Near-Optimal Dynamic Spectrum Allocation in Cellular Networks

Anand Prabhu Subramanian[1], Mahmoud Al-Ayyoub[1], Himanshu Gupta[1], Samir R. Das[1], Milind M. Buddhikot[2]
[1] Stony Brook University, NY, U.S.A.
[2] Alcatel-Lucent Bell Labs
{anandps, ayyoub, samir, hgupta}@cs.sunysb.edu, mbuddhikot@alcatel-lucent.com

***Abstract*—In this paper, we address the spectrum allocation problem in cellular networks under the coordinated dynamic spectrum access (CDSA) model. In this model, a centralized spectrum broker owns a part of the spectrum and issues dynamic spectrum leases to competing base stations in the region it controls. We consider a dynamic auction based approach where the base stations bid for channels depending on their demands. The broker allocates channels to them with an objective to maximize the overall revenue generated subject to wireless interference in the network.**

This problem is known to be NP-hard and has been addressed before in limited context. We address this problem in a very generic context where (i) interference in the network is modeled using pairwise and physical interference models and (ii) base stations can bid for heterogeneous channels of different width using generic bidding functions. We propose efficient approximation algorithms that give near optimal solutions with provable analytical bounds. Detailed simulation studies using randomly generated and real base station networks show that our algorithms scale very well for large network sizes.

I. Introduction

Usage of wireless spectrum by radio communication devices has long been governed by governmental regulatory authorities (e.g., FCC in USA or Ofcom in UK) that divide the spectrum into fixed size chunks to be used strictly for specific purposes, such as broadcast radio/TV, cellular/PCS services, wireless LAN/PANs, public safety related communication, etc. This allocation is very long-term and space-time invariant, and is often based on peak usage per provider. Many recent observations have shown that such long-term static allocation of spectrum introduces significant inefficiencies in utilization [1]. To improve spectrum utilization, there is a new policy trend [2] to make spectrum allocation more dynamic in both spatial and temporal dimensions and more responsive to end user demands.

There can be several different architectures for providing dynamic spectrum access (DSA) that can widely vary depending on the technological limitation and usage models. For example, one can consider a very flexible architecture (like in [3]) where individual nodes are envisioned to operate over a very wide band of spectrum (e.g., 0-3 GHz range). They can perform rapid spectrum sensing to identify spectrum holes and access

This work was partially supported by NSF grants CNS-0831791, CNS-0721665, CNS-0721455, CNS-0721701, IIS-0713186, CNS-0519734 and CNS-0423460. Milind M. Buddhikot was partially supported by the NSF grant CNS-0435348.

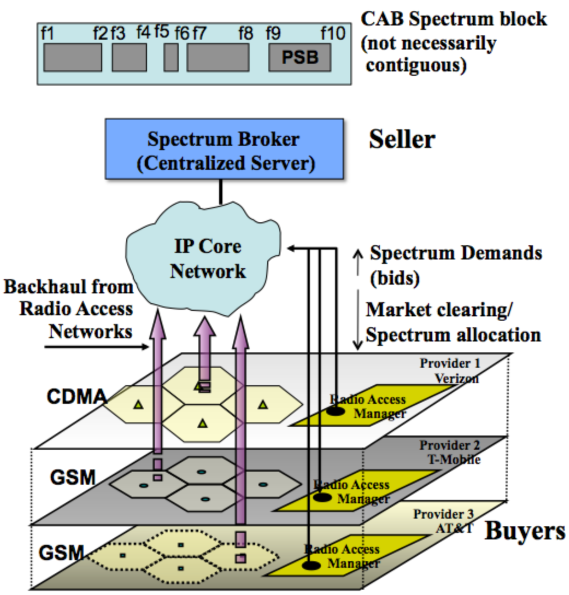


Fig. 1. Coordinated dynamic spectrum access architecture.

the free spectrum using a completely distributed coordination mechanism. This form of DSA (often referred to as cognitive radio) may be suitable for ad hoc on-demand networks, but unnecessarily complex for infrastructure based networks, such as the commercial cellular networks used by millions of end users worldwide. Buddhikot et al. [1], explored the application of a centralized architecture for dynamic spectrum access in cellular networks by introducing the *coordinated dynamic spectrum access* (CDSA) model which is much simpler and practical compared to fully distributed architectures. In the CDSA model (see Figure 1), there is a centralized entity known as the *spectrum broker* who owns a part of the spectrum called the *coordinated access band* (CAB) and dynamically allocates them to base stations in the region it controls. Indeed, centralized architectures [4–6] for dynamic spectrum access have gained a lot of interest in the research community due to their practicality and potential impact. However, success of the CDSA model hinges on the design of scalable and efficient spectrum brokers. We address this issue in this paper by designing efficient spectrum allocation algorithms that deliver near optimal solutions.

Problem Addressed. We consider a dynamic auction based approach to allocate spectrum to competing base stations. The centralized spectrum broker acts as the *seller* and the base stations (in the region controlled by the broker) act as the *buyers* of the CAB. The spectrum broker divides the CAB into channels (contiguous or non-contiguous blocks of frequency) and the base stations bid for these channels based on their spectrum demands. The base stations express their bids using a *bidding function* that specifies the price they are willing to pay for a given set of allocated channels. Periodically, the spectrum broker allocates available channels to the base stations (based on the received bids) under the "wireless interference constraint" such that the total revenue (total price paid by the base stations) is maximized. The above auction based approach allows the base stations to bid according to the spectrum demands, and the spectrum broker to maximize the revenue generated from allocation of spectrum.

The above spectrum allocation problem is known to be NP-hard and has been addressed before [5, 6] in limited contexts; e.g., [5] assumes unit-disk graphs to model interference between base stations, piece-wise linear bidding functions, and homogeneous set of non-overlapping channels, while [6] considers very primitive forms of bids and interference models. In contrast, we consider general network graphs and interference models (pairwise and physical), overlapping channels, and arbitrary non-complementary[1] bidding functions. For the above general context, we present approximation algorithms that deliver allocations with near-optimal revenue. Ours is the first work to address the above spectrum allocation problem in such general contexts.

Paper Organization: The rest of the paper is organized as follows. In Section II, we describe the system architecture of the CDSA model and give details of its components. In Section III and IV, we formally define and present efficient approximation algorithms for the spectrum allocation problem under pairwise and physical interference models, respectively. In Section V, we present detailed simulation results comparing performance of the proposed algorithms. Section VII concludes the paper with details about our future work.

II. System Architecture

In this section, we describe the reference system architecture (Figure 1) of our coordinated dynamic spectrum access model and give details of each important component of the model.

A. Spectrum Broker (Seller)

In the CDSA model, a centralized entity called the *spectrum broker* [1, 7] owns and coordinates access to the CAB in a given region and assigns short term spectrum leases to competing wireless service providers. Regulatory authorities like FCC can conduct one-time or long-term periodic auctions to give spectrum licenses to the broker on a regional basis. However, in contrast to existing cellular spectrum licenses, the spectrum broker can in turn grant spectrum leases that are for small geographical regions (e.g., per base station) and valid for short durations (e.g., tens of minutes) [7]. Such a spectrum lease gives the lessee exclusive rights to use the spectrum in the designated region for the duration of the lease without exceeding the maximum power limit. In this paper, we mainly address the challenge of how to assign these dynamic spectrum leases to various service providers and design fast and scalable spectrum allocation algorithms.

B. Base Stations or Nodes (Buyers)

The region under the control of the spectrum broker can have a number of base stations (also refered to as *nodes* in this article) owned by different Radio Infrastructure Providers (RIP). The Wireless Service Providers (WSP) (e.g., AT&T, Verizon) are customers of the RIPs and use their infrastructure to provide wireless services like voice, data etc. to end users. Each base station in the region can be used to operate different types of networks by the WSPs. For example, some base stations can be used to operate a GSM network, some for a CDMA or WCDMA network, and some for a WiMAX network. In a more general model, multiple types of networks can be operated on the same base station. Interference between different base stations depends on the location of the base stations, the frequency band used and the terrain propagation model [8]. We assume that the spectrum broker is aware of all the details of each base station in its region ranging from their exact location, and other characteristics like frequency range of operations, power levels, number of transmitters etc. It also knows the terrain propagation model in the region and can estimate the level of interference between base stations given their location and transmission power used. This knowledge forms part of essential inputs to our spectrum allocation algorithm.

C. Coordinated Access Band (Items Sold)

The portions of the spectrum that are highly underutilized or unused in spatial or temporal dimension qualify as prime candidates to be used as CAB. At the current time, good examples are Specialized Mobile Radio (SMR) (851-854/806-809 MHz, 861-866/816-821 MHz), public safety bands (PSB) (764-776, 794-806 MHz), and unused broadcast UHF TV channels (450-470 MHz,470-512 MHz (channels 14-20), 512-698 MHz (channels 21-51), 698-806 MHz (channels 52-69)). The CAB spectrum is to be shared between different cellular services with macro-cellular infrastructures. Some of the current technology examples that can use the above CAB spectrum are 1xRTT/1xEV-DO that use 1.25 MHz channels, GSM networks that use 200KHz channels, IS-136 legacy TDMA that uses 30 KHz channels, W-CDMA networks that use 5 MHz channels, WiMAX networks that can use 1.75 MHz to 20 MHz channels. Note that different technologies often provide different forms of services. Spectrum sharing between different services is advantageous as they provide the benefit of statistical multiplexing – the services use spectrum

[1] A bidding function is said to be *non-complementary* when it is defined on a set of items that do not complement each other. For example, the bid for choosing two items together should not be more than the sum of the bids for choosing the items individually.

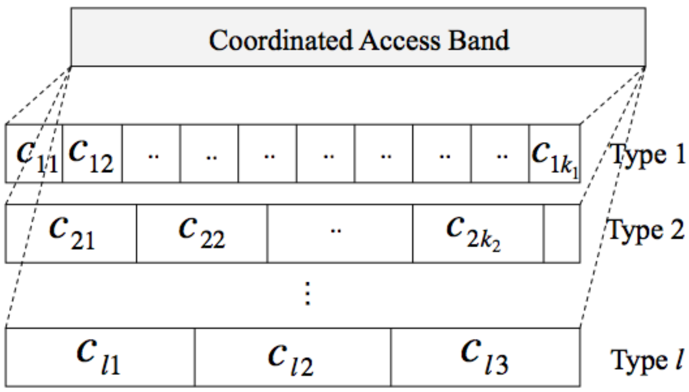


Fig. 2. Channels of different types (widths) in the CAB.

differently and have different load factors that vary largely on a spatio-temporal scale. It is also reasonable to use existing cellular bands (450 Mhz, 800 MHz or the 1.9 GHz band) as part of the CAB, giving a guaranteed access to incumbent WSPs who already hold licenses and on-demand access to other WSPs that do not conflict with the license holders.

Since different types of networks use channels of different widths, the spectrum broker has to make the decision on how to divide the available spectrum in to channels of different widths and allocate them to different base stations. In our model, we consider the spectrum broker divides the available spectrum into some *finite* number of channels for each type of network. *This channelization can be quite general.* For example, the spectrum broker may decide to create channels of varying width as shown in Figure 2; here, if W is the total width of CAB, then for i^{th} network/type, W/w_i non-overlapping channels of width w_i are created. Note that channels of different types that overlap with each other, cannot be assigned to the same or interfering base stations. Overlapping of channels makes the spectrum allocation problem very challenging compared to only using homogeneous channels as assumed in prior work [5, 6, 9].

D. Spectrum Demands, Bids, and Bidding Functions

The WSPs aggregate end user demands at each of the base station it operates and generate spectrum demands to the broker. Spectrum demand aggregation at each base station can be done using a predictive model based on historical traffic measurements or from end users' bandwidth inputs. The above demands are then used to generate bids for various combination of number of channels and channel types. In general, the bids are specified using a bidding function, which may be different for different base stations. Basically, the *bidding function* for any base station specifies the price the base station is willing to pay for each set of channels C. In general, the complexity of such a bidding function can be exponential in the number of channels, since the number of possible sets of channels is exponential. However, in simpler contexts, each base station may have a separate bidding function for each channel type, and the bidding function may specify a price depending on the *number* of channels of that type. In this article, we do not make any assumptions about the complexity of the bidding function,[2] unlike [5] where the authors assume the bidding functions to be piece-wise linear. The time complexity of our allocation algorithms is polynomial in the size of the bidding function.

III. Spectrum Allocation under Pairwise Interference

In this section, we address the spectrum allocation problem under the pairwise interference model. To give a formal definition of the problem, we need to define a few terms. Later, we design a greedy algorithm, and prove that it delivers a near-optimal spectrum allocation. We use the term *node* to refer to a base station.

Interference Graph (G_t). In the pairwise interference model, wireless interference between the nodes can be modeled using an *interference graph* which is defined as follows.

Definition 1: (Interference Graph G_t.) The interference graph $G_t = (V_t, E_t)$ is an undirected graph where each vertex represents a node and there is an edge $(u, v) \in E_t$ between u and v if the corresponding nodes "interfere". Two nodes are said to *interfere* if their corresponding "cells" (the surrounding regions they "cover" or are responsible for) intersect. Note that interfering nodes should not be allocated same or overlapping channels. □

Channel Graph (G_c). The overlapping nature of the channels in the CAB is modeled using a channel graph G_c defined below.

Definition 2: (Channel Graph G_c.) A channel graph $G_c = (V_c, E_c)$ is an undirected graph over channels as vertices, and there is an edge (c_i, c_j) between two channels c_i and c_j if they overlap with each other. For example, the channel graph corresponding to Figure 2 will have an edge between c_{23} and c_{15}. An empty (with no edges) channel graph means that the set of channels are mutually non-overlapping (as in the model of [5]). □

Valid Spectrum Allocation. Informally, our spectrum allocation problem is to allocate channels to nodes so as to maximize the total revenue (total price paid by the nodes). However, the allocation of channels should be done without violating the interference constraints. We formalize the above by defining a concept of valid spectrum allocation, in terms of conflicting node-channel pairs.

Definition 3: (Conflicting node-channel pairs.) Consider two node-channel pairs (u, c_i) and (v, c_j) where u and v are nodes and c_i and c_j are channels. The node-channel pairs (u, c_i) and (v, c_j) are said to be *conflicting* if the following is true: (i) $u = v$, or (u, v) is an edge in the interference graph, *and* (ii) $c_i = c_j$, or $(c_i, c_j) \in E_c$ (i.e., c_i and c_j overlap). □

Definition 4: ((Valid) Spectrum Allocation.) A *spectrum allocation* is a set of node-channel pairs, i.e., a spectrum

[2]Later, we do assume the bidding function to be non-complementary, to prove the performance guarantee of our designed algorithms.

allocation is a set

$$\{(u, c_i) | u \text{ is a node}, c_i \text{ is a channel}\}.$$

A spectrum allocation I is considered *valid* if no two node-channel pairs in I conflict with each other. □

Bidding Functions and Revenue. In general, a bidding function for a node u gives the price that u is willing to pay for a set of mutually non-overlapping channels. For the sake of simplicity, we use an equivalent notion of total revenue generated by a given valid spectrum allocation. Below, we formally define both the terms bidding functions and revenue.

Definition 5: (Bidding Function.) A *bidding function* F_u for a node u is a function $F_u : P(\mathcal{C}) \mapsto \mathbb{R}$, where $P(\mathcal{C})$ is the power set of all channels $\mathcal{C}$ and $\mathbb{R}$ is the set of real numbers. □

Definition 6: (Revenue $R(I)$). Given the bidding functions of nodes, the *revenue* generated by a *valid* spectrum allocation I is denoted by $R(I)$ and is defined as the sum of the prices paid by the nodes for the channels allocated to them by the spectrum allocation I. More formally,

$$R(I) = \sum_{u \in V_t} F_u(C_u),$$

where F_u is the bidding function of u, and $C_u = \{c_i | (u, c_i) \in I\}$ is the set of channels allocated to u by I. Revenue is defined only for valid spectrum allocations. □

Spectrum Allocation Problem. Based on the above definitions of valid spectrum allocation and revenue, the spectrum allocation problem under the pairwise interference model can be defined as follows.

Definition 7: (Spectrum Allocation Problem.) Given an interference graph, a channel graph, and the bidding functions for nodes, the spectrum allocation problem is to find a valid spectrum allocation I that maximizes the total revenue $R(I)$. □

Greedy Algorithm (GA). For the above spectrum allocation problem, we design a greedy algorithm that constructs a valid spectrum allocation by iteratively adding the "best" node-channel pair at each stage. We will show that such a greedy strategy results in a valid spectrum allocation with near-optimal revenue. A more formal description of our Greedy Algorithm for the spectrum allocation problem is as follows.

Let I be the valid spectrum allocation being constructed by the algorithm. Initially, $I = \phi$. In each iteration, the algorithm picks a node-channel pair (u, c_i) to add to I such that

- $I \cup (u, c_i)$ remains a valid spectrum allocation, and
- $R(I \cup \{(u, c_i)\}) - R(I)$, the "incremental revenue" is maximum (among all choices of node-channel pairs).

The algorithm terminates when I cannot be extended any further.

If n_t is the number of nodes, n_c is the number of channels, and Δ_t and Δ_c are the maximum vertex-degree in the interference and channel graphs respectively, then the overall time complexity of the above algorithm can be shown to be bounded by $O(n_t n_c \Delta_t \Delta_c log(n_t n_c))$ if we use a heap data structure to compute the maximum at each stage.

Performance Guarantee of GA. In the following theorem, we will show that the Greedy Algorithm returns a near-optimal valid spectrum allocation. However, to prove the approximation bound, we need to assume a certain "non-complementary" property of the revenue function. Given the bidding functions, we say that the revenue satisfies the *non-complementary property* if the following condition holds for any two valid spectrum allocations I_1 and I_2 such that $I_1 \cup I_2$ is also a valid spectrum allocation.

$$\max(R(I_1), R(I_2)) \leq R(I_1 \cup I_2) \leq R(I_1) + R(I_2) \quad (1)$$

Recall that revenue is only defined for *valid* spectrum allocations. It is easy to see that revenue is non-complementary if and only if the bidding functions are non-complementary. The above non-complementary property is commonly assumed in the auction literature [10], and signifies that no two valid spectrum allocations "complement" one another. More importantly, the above property entails that the incremental revenue of any particular node-channel pair never increases as the Greedy Algorithm progresses (i.e., with the selection of other nodes-channel pairs). Such a property is indeed essential for the Greedy Algorithm to have a bounded performance guarantee. Later in this section, we discuss scenarios where the non-complementary property may not be satisfied, but the Greedy Algorithm can still be modified appropriately to preserve the performance guarantee. We now prove the approximation ratio of the Greedy Algorithm, for non-complementary revenue functions.

Theorem 1: For a non-complementary revenue function, the above Greedy Algorithm (GA) returns a $(\delta_t(\Delta_c + 1) + 1)$-approximate valid spectrum allocation. Here, δ_t is the size of the maximum independent set in the neighborhood of any node in the interference graph, and Δ_c is the maximum degree of a vertex in the channel graph.

Proof: Let q_i be the i^{th} node-channel pair selected by GA in its i^{th} iteration, a_i be the corresponding incremental revenue of q_i, and m be the total number of iterations of GA for the given input. We use I_i to denote $\{q_1, q_2, ..., q_i\}$; thus, $a_i = R(I_i) - R(I_{i-1})$. Let O be the optimal solution and let O_m be the set of node-channel pairs in O that conflict with some pair in I_m. Below, we use the notation $R(I_1 | I_2)$ to denote $R(I_1 \cup I_2) - R(I_2)$ where I_1, I_2 and $I_1 \cup I_2$ are all some valid spectrum allocations.

We make the following three claims.

- For each $o \in O_m$, let $f(o)$ be the smallest integer such that o conflict with $q_{f(o)}$. Informally, selection of $q_{f(o)}$ by GA is the reason why o is not considered by GA for selection in later iterations. Note, by the greedy choice of q_l we have,

$$R(\{o\} | I_{f(o)-1}) < a_{f(o)} \quad (2)$$

- By definition of δ_t (the maximum size of an independent set in the neighborhood of any node in the interference graph), it is easy to see that the maximum number of mutually non-conflicting node-channel pairs that conflict with a particular q_i is $\delta_t(\Delta_c + 1)$. Here, Δ_c is the maximum degree of any vertex in the channel graph. Thus for any integer l, there are atmost $\delta_t(\Delta_c+1)$ node-channel pairs o in O_m such that $f(o) = l$. Thus we have,

$$\sum_{o \in O_m} a_{f(o)} \leq \delta_t(\Delta_c+1) \sum_{l=1}^{m} a_l = \delta_t(\Delta_c+1) R(I_m) \quad (3)$$

- Using induction on O_m, it is easy to shown that:[3]

$$R(O) \leq R(I_m) + R(O - O_m | I_m) + \sum_{o \in O_m} R(\{o\} | I_{f(o)-1}) \quad (4)$$

Without loss of generality, assume that the Greedy and optimal solutions are disjoint. Then, GA continues till $O_m = O$. For $O_m = O$, the above Equation 4 becomes:

$$R(O) \leq R(I_m) + \sum_{o \in O_m} R(\{o\} | I_{f(o)-1}). \quad (5)$$

Now using Equations 2 and 3 in the above Equation 5, we get

$$R(O) \leq (\delta_t(\Delta_c + 1) + 1) R(I_m),$$

yielding the approximation ratio.

■

Remarks. We make the following remarks, as special cases of the above result.

- If each base station has a circular cell of a fixed radius, then the the interference graph is a unit-disk graph, and δ_t is at most 5 [11]. In that case, the approximation ratio becomes $5\Delta_c + 6$.
- If we consider non-overlapping channels and a unit-disk interference graph, then the above theorem states that GA returns a 6-approximate solution. This is a direct generalization of the result in [5], for arbitrary revenue functions.

Handling Complementary Bidding Functions. We have so far assumed that the revenue function satisfies the non-complementary property (Equation 1). However, there may be scenarios where the bidding functions (and hence, the revenue function) may not satisfy the non-complementary property. In many such scenarios, our Greedy Algorithm (and similarly, the algorithms designed in next section) can be modified to ensure the approximation ratio.

For instance, consider the case where a node may bid for "groups" of channels, i.e., the node is willing to pay a high price for a group of channels C but bids zero price for any of the individual channels in C. More specifically, a node may pay a price of 100 units for channels 5 and 10 *together*, but pays nothing for either channel 5 or 10 individually. Such a bidding function is *complementary*. However, we can have our Greedy Algorithm handle the above case by creating *super-channels* corresponding to each such group of channels; we also have the set of super-channels include the singleton sets of individual channels. Then, the channel-interference graph is constructed over super-channels as vertices, and allocation is done in terms of such (node, super-channel) pairs. The modified GA, which selects a (node, super-channel) pair at each stage, still yields the same approximation ratio.

In a more general scenario of "packaged bids," a service provider (owning multiple nodes) may bid for a channel c_1 at a node u_1 only if a node u_2 is also allocated a channel c_2. In essence, a service provider may pay certain price for a *group* of node-channel pairs, but none for any individual pair. For the above case, the bidding functions cannot be defined independently for each node, but must be defined for each service provider (i.e., for the group of nodes owned by it). However, the revenue function can be easily computed from such bidding functions. But, the resulting revenue function is no longer non-complementary. Fortunately, our Greedy Algorithm can still be appropriately modified (by having it allocated in terms of groups of node-channel pairs) to handle the above case, while ensuring its approximation ratio.

For explicitly represented bidding functions (where a price is specified for *each* super-channel or package), GA still runs in time which is polynomial in the size of the input (including the representation of the bidding functions).

IV. Spectrum Allocation under Physical Interference

In this section, we use the physical interference model to capture interference between base stations in the network, and present two approximation alogrithms for the spectrum allocation problem in this context. We start by redefining the interference model and the concept of valid spectrum allocation.

Interference Model. Here, we assume that each node operates using a fixed transmission power P. In the physical interference model [12], a successful reception at a distance d from a node is possible, if the SINR at the receiver is greater than a threshold β. More formally, a successful reception for a node u is possible at a point p if and only if,

$$\frac{P/d_u^\alpha}{N + \sum_{v \in V'} P/d_v^\alpha} \geq \beta \quad (6)$$

where V' is the set of other nodes operating on the same (or overlapping) channel as u, d_x is the distance of the point p from a node x, N is the background noise, and α is the path loss exponent based on the terrain propagation model.

Communication Radius (r). The communication radius r is the maximum distance from a node u within which we *want* the SINR from u to be greater than β. Essentially, the above is based on the stipulation that a node's "cell" (surrounding region covered by a node) is a circular region of radius r around the node. In our context, the value of r can be arbitrarily large (but finite), since the approximation ratio and time complexity of our designed algorithms is independent

[3]Note that the base case is when $O_m = \phi$; in that case, the equation is true because $R(O) \leq R(O \cup I_m) = R(I_m) + R(O|I_m)$. For the inductive step, we just need to use the fact that $R(\{o\}|I_m) \leq R(\{o\}|I_{f(o)-1})$.

of r. Thus, *the concept of communication radius must not be looked upon as an assumption.*

Valid Spectrum Allocation. In the context of physical interference model, a spectrum allocation I is considered valid if it satisfies the following two conditions:

- For any node u, the set $\{c_i | (u, c_i) \in I\}$ of channels allocated to u consists of mutually non-overlapping channels.
- For a node-channel pair (u, c_i) in I, let V_{ui} denote the set of nodes that have been allocated in I some channel c_j that overlaps with c_i. More formally, let $V_{ui} = \{v | (v, c_j) \in I \text{ and } c_j = c_i \text{ or } (c_i, c_j) \in E_c\}$. Now, for I to be valid, for every (u, c_i) in I and every point p within a distance of r from u, SINR at p due to u should be greater than β; i.e, the following should hold:

$$\frac{P/d_u^{\alpha}}{N + \sum_{v \in V_{ui}} P/d_v^{\alpha}} \geq \beta$$

where d_x is the distance of node x from the point p.

Spectrum Allocation Problem. The spectrum allocation problem under the physical interference model is as follows. Given a set of nodes, the channel graph, and the bidding functions, the spectrum allocation problem is to select a valid spectrum allocation I that maximizes $R(I)$, the total revenue generated by I.

In the following paragraphs, we describe two greedy algorithms, viz. GAHT and GACP, for the spectrum allocation problem under physical interference model. The performance of both the algorithms depend on the interference model parameters (α and β). Our simulations results (presented in Section V) show that GAHT generates higher revenue than GACP for lower α and higher β values. In other cases, GACP generates higher revenue compared to GAHT. An appropriate choice between the two algorithms can be made depending on the actual values of α and β.

Greedy Algorithm Based on Hexagonal Tiling (GAHT). The basic idea of GAHT is as follows. We start with partitioning the entire region into hexagonal subregions of certain length, and color them using 3 colors such that no two adjacent subregions have the same color. See Figure 3. Then, we construct *three* valid spectrum allocations, one for each color. For a particular color k, we consider only k-colored hexagonal subregions and pick node-channel pairs iteratively (as in GA). However, we impose the condition that in any hexagonal subregion, only one node is assigned any particular channel; this condition ensures the validity of the spectrum allocation (as shown in Lemma 1). Finally, we pick the best of the three spectrum allocations (one for each color) thus constructed. We will prove that the above algorithm yields a near-optimal spectrum allocation (Theorem 2). A more formal description of GAHT is as follows.

- Partition the entire region into hexagonal subregions of side μr each, where r is the communication radius and μ is defined as:

$$\mu = 4 \sqrt[\alpha]{\frac{2\beta(3\alpha - 5)}{3(\alpha - 1)(\alpha - 2)}} \tag{7}$$

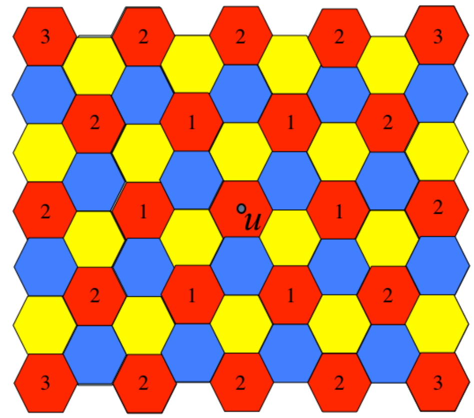


Fig. 3. Hexagonal subregions colored using three colors such that adjacent subregions have different colors. The red-colored subregions around the subregion containing node u have been partitioned into hierarchical levels; the numbers denote the hierarchical level. At l^{th} level, there are $6l$ red-colored subregions.

- Next, color the subregions using three colors, such that adjacent subregions are colored differently. See Figure 3.
- For each color k, consider only the k-colored subregions and construct a spectrum allocation I^k as below.
  - Initially $I^k = \phi$.
  - Pick a node-channel pair (u, c_i) with the highest incremental revenue to add to I^k such that (i) u lies in a k-colored subregion, (ii) No (v, c_j) already exists in I^k, such that v is in the same subregion as u and c_i overlaps with (or is same as) c_j.
  - Terminate when I^k cannot be extended any further.
- From the three spectrum allocations I^1, I^2, and I^3 thus constructed, pick the one that has the highest revenue and return it as the solution.

We now prove that each of the three spectrum allocations constructed by GAHT, the above algorithm, are valid. Intuitively, the spectrum allocations are valid because the total interference at any point p due to "far away" (in non-adjacent subregions) interferers is less than the signal received due to a node u in the subregion of p.

Lemma 1: GAHT returns a valid spectrum allocation.

Proof: Consider a node u in a subregion A of color k. As shown in Figure 3, partition all k-colored subregions surrounding A into hierarchical levels. The first level contains 6 subregions and each such subregion B is at a distance of μr from A; here, by distance between two subregions we mean that the distance between *any* point in B and *any* point in A is at least μr. Similarly, the second level contains 12 subregions at a distance of at least $2\sqrt{3}\mu r$ from A. In general, the l^{th} level contains $6l$ subregions at a distance of at least $\frac{\sqrt{3}}{2}(3l - 2)\mu r$ from A.

Now consider a point p within a distance of r from u. Let u be operating on a channel c_i. Recall that GAHT does not assign overlapping channels to any single node or to any two nodes in the same subregion. Thus, the total signal received at p due to all nodes possibly operating in c_i or an overlapping channel in the k-colored subregions other than A is at most:

$$\sum_{l=1}^{\infty} 6l \frac{P}{(((\frac{\sqrt{3}}{2}(3l-2))\mu - 1)r)^{\alpha}}$$
$$< \frac{2P(3\alpha-5)}{3(\alpha-1)(\alpha-2)(\frac{1}{4}\mu r)^{\alpha}}$$

Thus, ignoring the noise, the SINR of channel c_i at p due to u is at least:

$$\frac{\frac{P}{r^{\alpha}}}{\frac{2P(3\alpha-5)}{3(\alpha-1)(\alpha-2)(\frac{1}{4}\mu r)^{\alpha}}} = \beta.$$

The above follows from the value of μ in Equation 7. ∎

We now show that no valid spectrum allocation (in particular, the optimal) can have more than q base stations within a subregion allocated the same channel, where q is:

$$q = \frac{(2\mu-1)^{\alpha}}{\beta} \tag{8}$$

Lemma 2: No particular channel can be assigned to more than q nodes in any hexagonal subregion, by a valid spectrum allocation.

Proof: Assume the contrary, i.e., a valid spectrum allocation assigns a particular channel to $q+1$ nodes in same hexagonal subregion. Now consider a point p at a distance of r from one of these nodes u. Then, the SINR at p due to u is at most:

$$\frac{\frac{P}{r^{\alpha}}}{\frac{qP}{((2\mu-1)r)^{\alpha}}} < \beta.$$

∎

The above lemma can be used to show that GAHT returns a constant-factor approximate solution.

Theorem 2: GAHT returns a valid spectrum allocation whose revenue is at least $1/(3(q(\Delta_c+1)+1))$ of the optimal revenue, where Δ_c is the maximum degree of a vertex in the channel graph and q is as defined above in Equation 8. □

The proof of the above theorem is similar to that of Theorem 1, and is given in [13].

Greedy Physical Based on Circular Packing (GACP). We now present another algorithm (GACP) whose approximation proof is based on a circular packing argument. In short, GACP works by first constructing a "virtual" interference graph over the nodes. Two nodes are connected by a simple edge if the distance between them is less than $\mu' \cdot r$, where r is the communication radius and μ' is as defined below.

$$\mu' = \sqrt[\alpha-2]{\frac{2^{\alpha+2}(3\alpha-4)\beta}{(\sqrt[\alpha]{\beta}+1)^2(\alpha-1)(\alpha-2)}} \tag{9}$$

Then, GACP works exactly as the GA for the pairwise interference model, except that GACP uses the above constructed virtual interference graph as the pairwise interference graph. It can be shown that the resulting spectrum allocation is valid in the context of *physical* interference model, and its revenue is at least $1/(q'(\Delta_c+1)+1)$ of the optimal revenue possible, where Δ_c is the maximum degree of a vertex in the channel graph and q' is as defined below.

$$q' = \frac{4\mu'^2 + 4\mu'(\sqrt[\alpha]{\beta}+1) + (\sqrt[\alpha]{\beta}+1)^2}{(\sqrt[\alpha]{\beta}+1)^2} \tag{10}$$

Theorem 3: GACP returns a valid spectrum allocation under physical interference model whose revenue is at least $1/(q'(\Delta_c+1)+1)$ of the optimal revenue. □

The validity of spectrum allocation can be proved using a circle-packing technique following a similar argument as in the proof of Lemma 1. To prove the approximation ratio, we need to show that the size of the maximum independent set in the neighborhood of any node in the "virtual" interference graph used in the GACP algorithm is q'. We can show this using a simple packing argument. Consider a circle of radius $\mu' r$ around any base station. Then the maximum number of non-overlapping circles of radius $(\sqrt[\alpha]{\beta}+1)/2$ that overlap with this circle is given by Equation 10. A detailed version of the proof for the above theorem is given in [13].

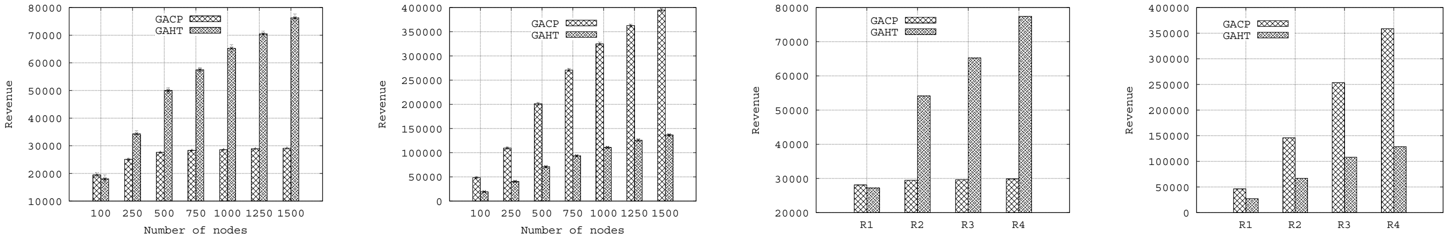
V. Simulation

In this section, we present detailed simulation results comparing the performance of the proposed algorithms. First, we compare the performance of the two greedy algorithms under the physical interference model (GAHT and GACP) for different network topologies and interference model parameters (α and β). Next we examine how well a spectrum allocation returned by the greedy algorithm (GA) under the pairwise interference model work under the physical interference model. We start by describing the simulation parameters and then present the results.

Network Topology. In order to examine the impact of network topology, we consider two types of networks.

- *Random Networks:* We consider a fixed area of 1000×1000 units and randomly place base stations within this area. We vary the network density by changing the number of base stations from 100 to 1500. We assume a communication radius r of 25 units in this scenario.
- *Real Networks:* We use locations of real cellular base stations availalbe in FCC public GIS database [14] and choose base stations deployed in 4 different regions of increasing size and number of base stations.
  - R1 - 843 base stations in the state of MA
  - R2 - 2412 base stations in New England area (MA, ME, NH, VT, RI, CT)
  - R3 - 4467 base stations in New England and New York
  - R4 - 8618 base stations in North East USA (New England, NY, NJ, PA)

  Here the regions are progressively supersets of the previous ones. We assume the communication radius r to be 25 meters in this scenario.



(a) Random Networks, $\alpha = 2.5$ (b) Random Networks, $\alpha = 4$ (c) Real Networks, $\alpha = 2.5$ (d) Real Networks, $\alpha = 4$

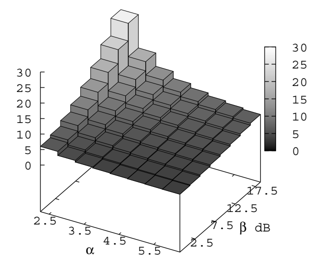
Fig. 4. Comparison of overall revenue generated by GAHT and GACP algorithms for various network topologies.

CAB. We consider a CAB with a bandwidth of 300 MHz and assume that each base station in the region can operate one or more of the following types of networks: GSM (200 KHz), CDMA (1.25 Mhz) and W-CDMA (5 Mhz). We assume the CAB is divided in to channels of different types as described in Figure 2 and so we have 1500 GSM channels, 240 CDMA channels and 60 W-CDMA channels in total.

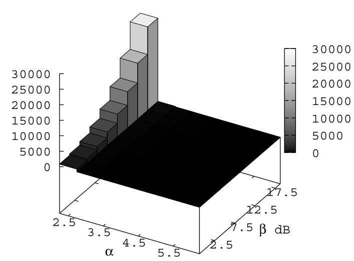
Bidding Functions. We generate bidding functions for each base station as follows. First, we randomly assign the number (between one and three) of types of channels/networks operated at the base station. For each network type, we generate a separate bidding function as follows. Let m be the number of channels in the given network type. We generate m random numbers from a predetermine range; let the generated numbers in the non-increasing order be: $\{p_1, p_2, p_3, \ldots, p_m\}$. Now, for any set of channels of size k, we assign the bidding price to be $\sum_{i=1}^{i=k} p_i$. Such a bidding function essentially gives higher value to channels that are allocated earlier. In terms of price ranges, for GSM networks, we generate prices from the interval 1–20, for CDMA networks we use the interval 1–125 and for W-CDMA networks we use the interval of 1–500. The intervals are chosen as above so that channels that have higher width (e.g. W-CDMA compared to GSM) are valued at a higher price by each base station.

A. Comparing GAHT and GACP

In our first set of experiments, we compare the performance of the two greedy algorithms under the physical interference model. In Figure 4, we show the revenue generated using the two algorithms for different network sizes in both random and real network topologies for two different values of α and a fixed value of β = 5 dB. For the random network case (see Figures 4(a) and 4(b)), we see that in both algorithms, the revenue generated increases with the network size. This is mainly due to the non-increasing nature of the bidding functions. So with more nodes, the spectrum broker tries to allocate channels to nodes that are willing to pay more (those nodes with less number of channels allocated) and thereby generating higher revenue. We also see when the network size is small (about 100), the difference between revenue generated by both algorithms is small. As the network size increases, the difference becomes increasingly high. We see similar behavior in the real network scenarios (see Figures 4(c) and 4(d)). It is interesting to note that when $\alpha = 2.5$, the revenue generated by GACP is much poorer than that of GAHT, and when $\alpha = 4$ the scenario is exactly opposite. This is mainly due to dependence of size of the hexagonal subregion with side μr (used in GAHT) and the circular region of radius $\mu' r$ (used in GACP) on α and β values.



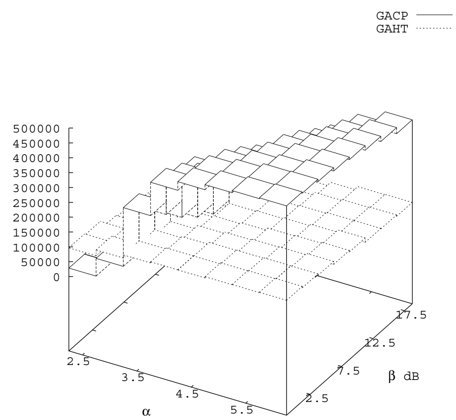
(a) μ used in GAHT algorithm



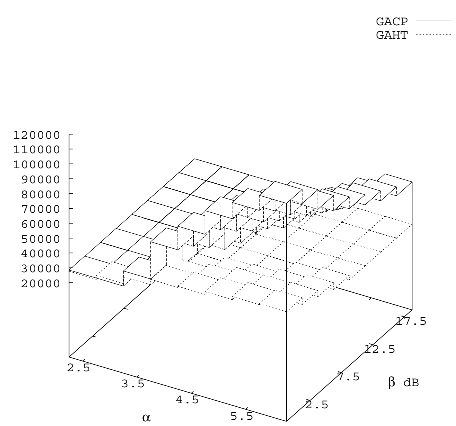
(b) μ' used in GACP algorithm

Fig. 5. μ and μ' values for different values of α and β.

To understand this behaviour clearly, we show the values of μ and μ' for various values of α and β in Figure 5. We see that



(a) Random Networks



(b) Real Networks

Fig. 6. Comparison of overall revenue generated by GAHT and GACP algorithms for varying α and β values.

μ' is very large compared to μ for small α. Since we assume nodes within a distance of $\mu' r$ interfere with each other in GACP, the revenue due to channel reuse is much less. The variation of μ and μ' due to β is relatively small compared to the variation due to α. As α increases and β decreases, μ and μ' become small and more channel reuse is possible.

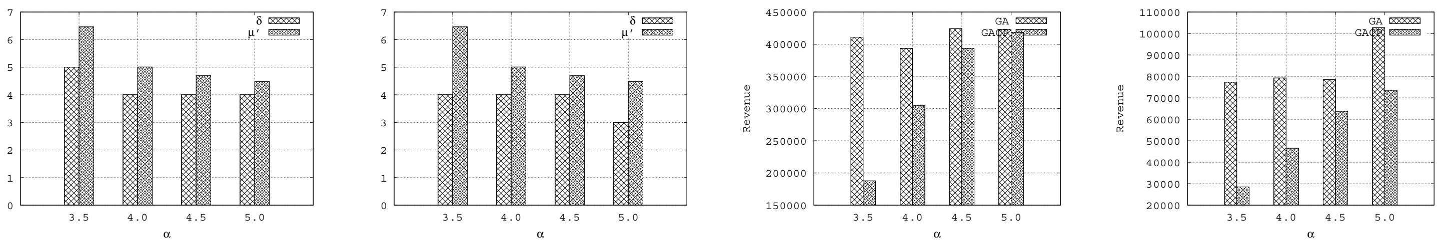
We show the revenue generated in a 1000 node random network and a real network (R1) with varying values of α and β in Figure 6. We see that when $\alpha < 3.5$ and $\beta > 7.5$ dB, GAHT generates higher revenue compared to GACP and it is the opposite in other cases. We conclude that the performance of both the greedy algorithms mainly depends on the interference model parameters and an appropriate choice can be made depending on the actual values of α and β.

B. Comparing GA and GACP

In this section, we compare the performance of the greedy algorithm (GA) under the pairwise interference model and GACP. Note that both algorithms are based on the same technique except the way the interference graphs are constructed. In GA, two nodes interfere when they are within a distance d. In GACP, two nodes interfere when they are within a distance $\mu' r$. Assume $d = \delta r$ for some δ. In order to do a fair comparison between GA and GACP, we need to make sure that the spectrum allocation obtained using GA for some value of δr is actually "valid" in the physical interference model. We do this in the following manner. We increase the value of δ from 2 to 8 and compute a spectrum allocation using GA. For each value of δ, we use the spectrum allocation obtained to test whether the SINR at any point within a distance of r from any base station is less than β. Since it is computationally hard to do this test in the infinitely large number of points in the region, we only check on 8 selected point on the circumference of the circle of radius r around each base station.[4] If the SINR constraint is satisfied for all base stations, then we use the revenue generated by this spectrum allocation to compare with the revenue generated by the GACP algorithm. Otherwise we increase the value of δ and repeat the above procedure. We repeat this test for different values of α for a 1000 node random network and one real network (R1). The value of β is fixed at 5 dB in all experiments.

In Figure 7(a) and 7(b), we show the values of δ compared to μ' when the spectrum allocation obtained using GA is valid under the physical interference model. In the case of random networks, δ is smaller than μ' by about 17% on average. In the case of real networks, we see its is smaller by about 26% on average. Note that when δ is same as μ', the revenue generated by GA and GACP should be same as both algorithms are similar. Due to the smaller value of δ, GA can exploit much higher spatial reuse of spectrum and generate more revenue. The revenue generated by both the algorithms are shown in Figures 7(c) and 7(d). The difference between the two algorithms is higher in the case of real networks due

[4]It can be shown easily that if the SINR constraint is satisfied on the circumference of the circle, then it will be satisfied at any point within the circle.



(a) Random Networks, δ Vs. μ'

(b) Real Networks, δ Vs. μ'

(c) Random Networks, Revenue Comparison

(d) Real Networks, Revenue Comparison

Fig. 7. Comparing performance of GAPW and GACP algorithms.

to the larger difference between δ and μ' compared to the random network case. This clearly demonstrates that while the pairwise interference model is simplistic, it can be used to generate efficient spectrum allocations (with an appropriate choice of d) which are valid in the real physical interference model. But in order to prove good theoretical properties for the spectrum allocation algorithm under the physical interference model, we need to use the definition of μ'.

VI. Related Work

Traditional auctions. Auctions have been traditionally used for efficiently allocating scarce resources [15–17]. The auctioneer can maximize his revenue by selling the goods to buyers who are willing to pay the most. At the same time, the buyers also get benefited as auctions tend to assign items to buyers who need them the most based on their valuation. Some examples where auction systems have been successfully used include energy markets [16], treasury bonds [17], and selling commercial goods online [15]. In general, the goods on sale can either be a single item [10], bundle of multiple units of single items [10, 18] or bundles of multiple units of multi-items [19, 20] and the complexity of the auction mechanisms increase in this order.

The spectrum allocation problem in the CDSA model differs from traditional periodic sealed bid multi-unit auctions in the following two important aspects. First, in a conventional multi-unit auction, every buyer competes with every other buyers participating in the auction. In the problem considered in this work, there is a network of base stations, and each base station competes only with other base stations with which they interfere. This increases the complexity of the auction problem significantly as the way in which the base stations interfere depends on external constraints that include complexities such as radio propagation model, frequency used and transceiver design etc. While traditional multi-unit auctions can be solved optimally in polynomial time, this class of auction problems are known to be NP-hard even when specific restricted class of bidding functions [5] are used. The second major difference is the overlapping nature of channels of different types which puts an additional constraint in assigning channels to base stations. In traditional auctions, any item can be assigned to any buyer that is not true here.

Spectrum allocation without revenue models. Spectrum allocation algorithms, both centralized [7, 21] and distributed [9, 22] in the context of dynamic spectrum access networks have been proposed previously without any revenue model. In these works, the authors propose algorithms to allocate spectrum to different nodes thereby optimizing one or more network properties like network interference or network capacity. All these algorithms only consider pairwise interference models and do not consider heterogeneous channels of varying widths that can overlap. In context of ad hoc networks, Yuan et al. in [23] propose centralized and distributed allocation of variable width frequency blocks to nodes in the network in a time-slotted fashion. Our work differs from theirs in two main aspects. We have a general revenue model associated with the channels and try to maximize the overall revenue while they optimize a proportionally fair throughput metric. The second difference is that we propose efficient algorithms using both pairwise and physical interference models while they use only pairwise interference model. In [24], the authors propose a spectrum allocation algorithm intergrated with interference-aware statistical admission control. Here also, they do not consider any revenue model and use only pair-wise interference model to capture interference between access points.

Revenue maximizing spectrum allocation. Two previous works that are directly related to our work are [6] and [5]. In [6], Sengupta et al. formulate the spectrum allocation problem as a modification of the knapsack problem. Here they assume a very primitive revenue model where they consider a constant price for each channel and specify spectrum demands as a fixed number of channels. The spectrum broker should either allocate all channels demanded or it cannot allocate any channel. This kind of spectrum demand is too restrictive to support efficient allocation. Also, they only consider homogeneous type of channels. In [5], Gandhi et al. propose solutions for the revenue maximizing spectrum allocation problem under pair-wise interference model. Here, the authors only consider a specific class of revenue function which is piece-wise linear in nature and use only homogeneous channels. Their algorithms cannot be extended to work for any general class of revenue functions and heterogeneous types of overlapping channels as we consider here. If we consider homogeneous channels only, our approximation is still better considering the fact we can solve the problem for any general revenue function. In addition, we address the spectrum allocation problem under physical interference model.

VII. Conclusion and Future Work

In this paper, we proposed efficient approximation algorithms that give near optimal solutions for the spectrum allocation problem in cellular network under the coordinated dynamic spectrum access model. We addressed the spectrum allocation problem in a very general context where (i) interference in the network is modeled using pairwise and physical interference models and (ii) base stations can bid for heterogeneous channels of different width using generic bidding functions. Ours is the first work to propose efficient solutions in such general context for this problem. For the specific case of non-overlapping channels and a unit-disk interference graph, our greedy algorithm GA returns a 6-approximate solution which is a direct generalization of the results in [5] for arbitrary revenue functions. Our simulations studies show that the proposed algorithms scale very well for large network topologies. Among the two algorithms proposed for the physical interference model, we see their performance primarily depends on the interference model parameters and the appropriate algorithm can be chosen based on the actual value of α and β. We also see that the simple pairwise interference model can be used to come up with efficient spectrum allocations that are valid under the physical interference model by appropriately choosing the interference region around the base stations.

As part of our future work, we plan to develop techniques to model the dynamics of real spectrum demands and bids using realistic population data and user calling patterns and study the scalability of our approaches. We also plan to address the problem of joint power and spectrum allocation in this context.

References

[1] M. M. Buddhikot, P. Kolodzy, S. Miller, K. Ryan, and J. Evans, "DIMSUMnet: New Directions in Wireless Networking Using Coordinated Dynamic Spectrum Access," in *Proceedings of IEEE WoWMoM 2005*, June 2005.

[2] Spectrum Policy Task Force, "Spectrum Policy Task Force Report," *Federal Communications Commission ET Docket 02-135*, 2002.

[3] DARPA XG-Program. [Online]. Available: http://www.darpa.mil/ato/programs/xg/

[4] V. Brik, E. Rozner, S. Banerjee, and P. Bahl, "DSAP: A Protocol for Coordinated Spectrum Access," in *Proceedings of IEEE DySPAN*, Baltimore,Maryland, November 2005.

[5] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri, "A General Framework for Clearing Auction of Wireless Spectrum," in *Second IEEE International Symposium on New Directions in Dynamic Spectrum Access Networks (DySPAN)*, Dublin, Ireland, April 2007.

[6] S. Sengupta, M. Chatterjee, and S. Ganguly, "An economic framework for spectrum allocation and service pricing with competitive wireless service providers," in *Second IEEE International Symposium on New Directions in Dynamic Spectrum Access Networks (DySPAN)*, Dublin, Ireland, April 2007.

[7] M. M. Buddhikot and K. Ryan, "Spectrum Management in Coordinated Dynamic Spectrum Access Based Cellular Networks," in *Proceedings of IEEE DySPAN*, Baltimore,Maryland, November 2005.

[8] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[9] L. Cao and H. Zheng, "On the Efficiency and Complexity of Distributed Spectrum Allocation," in *Second International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*, August 2007.

[10] V. Krishna, *Auction Theory*. Academic Press, 2002.

[11] M. V. Marathe, H. Breu, H. B. Hunt III, S. S. Ravi, and D. J. Rosenkrantz, "Simple Heuristics for Unit Disk Graphs," *Networks*, vol. 25, pp. 59–68, 1995.

[12] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, 2000.

[13] A. P. Subramanian, M. Al-Ayyoub, H. Gupt, S. R. Das, and M. M. Buddhikot, "Near-Optimal Dynamic Spectrum Allocation in Cellular Networks," Computer Science Department, Stony Brook University, Stony Brook, NY, Tech. Rep., August 2008.

[14] FCC Geographic Information Systems. [Online]. Available: http://wireless.fcc.gov/geographic/

[15] EBay, http://www.ebay.com.

[16] S. Borenstein, "The Trouble with Electricity Markets: Understanding California's Restructuring Disaster," *Journal of Economic Perspectives*, vol. 16, no. 1, 2002.

[17] K. Binmore and J. Swierzbinski, "Treasury auctions: Uniform or discriminatory?" *Review of Economic Design*, vol. 5, no. 4, December 2000.

[18] EBay - Multi Unit Auctions, http://pages.ebay.com/help/buy/buyer-multiple.html.

[19] P. Cramton, Y. Shoham, and R. Steinberg, *Combinatorial Auctions*. MIT Press, 2006.

[20] K. Binmore and J. Swierzbinski, "Combinatorial Auction: A Survey," *INFORMS Journal on Computing*, vol. 3, 2003.

[21] A. P. Subramanian, H. Gupta, S. R. Das, and M. M. Buddhikot, "Fast Spectrum Allocation in Coordinated Dynamic Spectrum Access Based Cellular Networks," in *Second IEEE International Symposium on New Directions in Dynamic Spectrum Access Networks (DySPAN)*, Dublin, Ireland, April 2007.

[22] J. Zhao, H. Zheng, and G. Yang, "Distributed Coordination in Dynamic Spectrum Allocation Networks," in *First IEEE International Symposium on New Directions in Dynamic Spectrum Access Networks (DySPAN)*, November 2005.

[23] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu, "Allocating Dynamic Time-Spectrum Blocks in Cognitive Radio Networks," in *Proceedings of ACM MobiHoc*, Montreal, Canada, September 2007.

[24] L. Cao and H. Zheng, "Stable and Efficient Spectrum Access in Next Generation Dynamic Spectrum Networks," in *IEEE International Conference on Computer Communications (INFOCOM)*, Phoenix, Arizona, April 2008.