The Effect of Interference on the Capacity of Multi-hop Wireless Networks

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I Extended Abstract

We study the effect of interference on the achievable rate region in multi-hop wireless networks, under three increasingly constraining interference models and characterize the network throughput in each of these models. We express the necessary conditions for achievability in these three models as linear constraints, and exploit the structure of these constraints to develop efficient fully polynomial time approximation algorithms that route end-to-end flows between multiple source destination pairs. This solution is then used to construct feasible schedules using approximate set coloring algorithms. The techniques developed in this paper are applicable to a wide range of problems, including capacity problems, arising in multi-hop networks with interference constraints.

We are interested in determining if some given end-to-end flows are achievable in the network under different interference models. We consider three different interference models in this paper.

***Primary Conflict Avoidance (PCA)*: In this model, we assume that transmissions between two distinct pairs of nodes in the network never results in a collision. Collisions occur only when a node attempts to send to or receive transmission from two different nodes simultaneously.**

***Receiver Conflict Avoidance (RCA)*: If there is only one channel shared between all nodes in the network, then, for a successful transmission, all other nodes that can potentially interfere with the receiver must remain silent.**

***Transmitter-Receiver Conflict Avoidance (TRCA)*: In this model, for a successful transmission to occur, the neighbors of the transmitter and the receiver have to refrain from both transmitting and receiving for the duration of the transmission. This is due to the fact that the physical layer expects the transmitter to feedback from the receiver to ensure a successful transmission.**

The nodes and links in the network are represented with a directed graph $G = (V, E)$ where V represents the set of nodes in the network and E the set of directed edges $\{e : t(e) \to r(e)\}$ (links from potential transmitter $t(e)$ to receiver $r(e)$) in the network. We say that a link e is *active* when there is a transmission from $t(e)$ to $r(e)$. A node cannot be simultaneously in communication with more than one node. Link e can transmit data at $c(e)$ bits/second. If a link is active for one time slot of length τ, then $\tau c(e)$ bits will be transmitted from $t(e)$ to $r(e)$. If $f(e)$ is the achieved bit rate (in bits/second) along link e, then the fraction of time link e is active is the *link utilization*, $x(e) = f(e)/c(e)$.

The necessary and sufficient constraints for a link flow vector to be achievable are given in Table 1, where $y_e(t)$ is 1 if link e is active at time t, and 0 otherwise. $f(e) = \frac{c(e)\sum_{t\leq T} y_e^t}{T}$.

Interference Model	LP: Continuous Variables (Necessary Conditions)
PCA	$\sum_{e\in E(v)} x(e) \leq 1, \forall v \in V$
RCA	$\sum_{e\in E(v)} x(e) \leq 1, \forall v \in V$ $\sum_{e'\in E_{out}(t(e))\cup E_{in}(r(e))} x(e') \leq 1,\ \forall e \in E$
TRCA	$\sum_{e'\in E(t(e))\cup E(r(e))} x(e') \leq 1,\ \forall e \in E$

Table 1: Constraints for Three Interference Models

Lemma 1 ***Let $S_1, S_2, \ldots, S_K$ represent sets of link such that no two links in a set can be active in the same time slot. Let* f *represent a link flow vector, where $f(e)$ represents the flow on link e. If* f *does not satisfy the following inequalities***

$$\sum_{e\in S_k} x(e) = \sum_{e\in S_k} \frac{f(e)}{c(e)} \leq 1, \qquad \forall k \tag{1}$$

then* f *is not schedulable.

Given a link flow vector f that satisfies the necessary conditions in Lemma 4, we want to determine if it is achievable. If a link flow vector f satisfies Lemma 1, we see if it is achievable by generating a schedule that achieves f.

Definition 1 ***The* set-chromatic index *of a multi-graph is defined as the minimum number of colors needed to color the links of the multi-graph such that* no two links in the same set S_k have the same color.**

Lemma 2 ***Let L represent the set-chromatic index of the scheduling multi-graph. Then***

$$\frac{f(e)}{L\tau} \quad e \in E$$

is an achievable flow.

- **If the vector f does not satisfy the necessary conditions, then output f is infeasible.**
- **Determine τ such that $x(e)/\tau$ is integral. $w(e) = x(e)/\tau$.**
- **Construct the scheduling multi-graph and determine its set-chromatic index L.**
- **If $L\tau \leq 1$, then f is achievable.**

We use greedy coloring to achieve reasonably good performance for all classes of interference considered here.

Theorem 3 ***Given a graph*** $G = (V, E)$***, with link speed*** $c(e)$ ***associated with link*** $e \in E$***,*** Q ***source destination pairs*** $(s(q), d(q))$ ***for*** $q = 1, 2, \ldots Q$ ***with a desired flow rate*** $r(q)$ ***between*** $s(q)$ ***and*** $d(q)$***, let*** $y_q(e)$ ***be the flow on link*** e ***that belongs to the end-to-end flow*** q***. A necessary condition for rate vector*** r ***to be achievable is the existence of link flows*** $y_q(e), \forall q, e$ ***that satisfies the following constraints.***

$$\begin{aligned}\sum_{e:t(e)=s(q)} y_q(e) &= r(q) \\ \sum_{e \in E_{in}(v)} y_q(e) &= \sum_{e \in E_{out}(v)} y_q(e) \;\; v \neq s(q), d(q) \;\forall k \\ \sum_{e \in S_k} \frac{\sum_{q \leq Q} y_q(e)}{c(e)} &\leq 1 \;\; \forall k.\end{aligned}$$

The first constraint ensures the end-to-end rate is met. The second constraint maintains flow balance at intermediate nodes in the network for each end-to-end flow. The third constraint is the interference constraint.

An alternate formulation of the above conditions can be given in an arc-path formulation. Let $\mathcal{P}_q$ **represent the set of paths for the source-destination pair** q**. Consider a path** $P \in \mathcal{P}_q$**. Let** $y(P)$ **be the amount of flow sent on that path. This path leads from** $s(q)$ **to** $d(q)$**. From the demand requirements, note that**

$$\sum_{P \in \mathcal{P}_q} y(P) = r(q) \quad \forall q.$$

The total amount of flow on link e**, represented by** $f(e)$ **is given by**

$$f(e) = \sum_q \sum_{P \in \mathcal{P}_q : e \in P} y(P).$$

Let $I(P, k)$ **be the set of links on path** P **incident on the set** S_k**, i.e,**

$$I(P, k) = \{e : e \in P\} \cap \{e : e \in S_k\}.$$

The amount of flow permitted by set S_k **on path** P **is given by**

$$F(P, k) = \sum_{e \in I(P,k)} c(e)^{-1}.$$

The flow that can be sent on path P **denoted by** $F(P) = \min_{k \in K} F(P, k)$**. We use** $F()$ **to represent flow on paths and** $f()$ **to represent flow on links. Note that there can be an exponential number of paths between two given nodes in the network. Therefore, the path-arc formulation has an exponential number of variables. The necessary conditions for a rate vector** r **to be achievable is given by**

$$\begin{aligned}\sum_{P \in \mathcal{P}_q} y(P) &= r(q) \\ \sum_{e \in S_k} \frac{\sum_k \sum_{P \in \mathcal{P}_q : P \ni e} y(P)}{c(e)} &\leq 1, \quad \forall k.\end{aligned}$$

Given a rate vector r**, the strategy then is to solve for the** y **variables that satisfies the necessary conditions.**

Instead of solving the feasibility problem directly, we write it in the form of a concurrent flow problem. In the concurrent flow problem, the desired rate vector is scaled and the objective is to determine the maximum scaling factor that still satisfies the necessary conditions.

A Solving the Linear Programming Problem

We first write the feasibility problem as a concurrent flow problem and then use a primal-dual algorithm to solve the linear programming problem.

$$\max \lambda$$

$$\begin{aligned}\sum_{e \in S_k} \frac{\sum_q \sum_{P \in \mathcal{P}_q : P \ni e} y(P)}{c(e)} &\leq 1 \quad \forall k \\ \sum_{P \in \mathcal{P}_q} y(P) &= \lambda\, d(q) \quad \forall q \\ y(P) &\geq 0 \quad \forall P \in \mathcal{P}_q \;\; \forall q.\end{aligned}$$

Let λ^* **be the optimal solution to the linear programming problem above.** λ^* **represents the maximum scaling factor by which flows can be scaled up, and still satisfy the necessary constraints. Therefore, if** $\lambda^* < 1$**, then the rate vector is not feasible. The largest flow vector that still satisfies the necessary constraints is** $\lambda^* \mathbf{r}$ **which is given by the optimal link flow vector** $\mathbf{y}^*$**. Applying Lemma 2 to this flow, we get a schedule for a flow vector** $\frac{\lambda^* \mathbf{r}}{L^* \tau}$**, where** L^* **is the set-chromatic index of the scheduling multi-graph generated by** $\mathbf{y}^*$**. For this achievable flow to be at least r, we need** $\lambda^* \geq L^* \tau$**.**

Theorem 4 ***If*** $\lambda^* < 1$***, then*** $\mathbf{r}$ ***is not schedulable. If*** $\lambda^* \geq L^* \tau$***, then there exists a schedule for the flow*** $\frac{\lambda^* \mathbf{r}}{L^* \tau}$***, and hence for*** $\mathbf{r}$***. If*** $1 \leq \lambda^* \leq \frac{\lambda^* \mathbf{r}}{L^* \tau}$***, then it is not known whether there exists a valid schedule for*** $\mathbf{r}$***.***

The dual to this problem assigns a weight $\alpha(k)$ **to each set** S_k **in the network.**

$$\min \sum_k \alpha(k)$$

$$\begin{aligned}\sum_{e \in P} \frac{\sum_{k : e \in S_k} \alpha(k)}{c(e)} &\geq z(q) \quad \forall P \in \mathcal{P}_q \;\; \forall q \\ \sum_{q=1}^{Q} d(q) z(q) &\geq 1 \\ \alpha(k) &\geq 0 \quad \forall k\end{aligned}$$

References

[1] M. Kodialam and T. Nandagopal, "Characterizing the Achievable Rates in Multihop Wireless Networks," *ACM Mobicom*, 2003.

[2] M. Kodialam and T. Nandagopal, "Interference and Achievable Rates in Multihop Wireless Networks," *Bell Labs Technical Report*, 2003.