Dynamic Spectrum Access in DTV Whitespaces: Design Rules, Architecture and Algorithms

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ABSTRACT

In November 2008, the FCC ruled that the digital TV whitespaces be used for unlicensed access. This is an exciting development because DTV whitespaces are in the low frequency range (50-698 MHz) compared to typical cellular and ISM bands, thus resulting in much better propagation characteristics and much higher spectral efficiencies. The FCC has also mandated certain guidelines for short range unlicensed access, so as to avoid any interference to DTV receivers. We consider the problem of Wi-Fi like access (popularly referred to as Wi-Fi 2.0) for enterprizes. We assume that the access points and client devices are equipped with cognitive radios, i.e., they can adaptively choose the center frequency, bandwidth and power of operation. The access points can be equipped with one or more radios. In this paper, we layout the design of a complete system that *(i)* does not violate the FCC mandate, *(ii)* dynamically assigns center frequency and bandwidth to each access point based on their demands and *(iii)* squeezes the maximum efficiency from the available spectrum. This problem is far more general than prior work that investigated dynamic spectrum allocation in cellular and ISM bands, due to the non-homogeneous nature of the whitespaces, i.e., different whitespace widths in different parts of the spectrum and the large range of frequency bands with different propagation characteristics. This calls for a more holistic approach to system design that also accounts for frequency dependent propagation characteristics and radio frontend characteristics. In this paper, we first propose design rules for holistic system design. We then describe an architecture derived from our design rules. Finally we propose demand based dynamic spectrum allocation algorithms with provable worst case guarantees. We provide simulation results showing that *(i)* the performance of our algorithm is within 94% of the optimal in typical settings and *(ii)* and the DTV whitespaces can provide significantly higher data rates compared to the 2.4GHz ISM band. Our approach is general enough for designing any system with access to a wide range of spectrum.

1. INTRODUCTION

Across the world, countries are migrating from analog to digital television broadcasts. For example, in the US, this transition happened on June 12, 2009; while in the UK, this transition is slated to happen in a phased manner from 2008 to 2012. In analog transmission, each TV channel uses a 6 MHz slice of bandwidth, but digital transmissions have the ability to pack four "programs" in one 6 MHz channel. Thus, this analog-to-digital transition frees up a substantial amount of television spectrum that was previously used by analog transmitters. The newly freed up spectrum (along with other slices of unused spectrum in the 50-700 MHz (channels 21-51) television band is known as DTV white-space (DTV-WS).

Signals in the DTV spectrum propagate over long distances and penetrate through obstacles more easily. Also, according to a recent study by Free Press and NAF [14], depending on the local market, somewhere between 100-250 MHz of DTV-WS will be made available. The large amount of spectrum and its superior propagation characteristics make the DTV-WS a highly attractive proposition for wireless broadband deployment and usage.

In November 2008, in its second report and order [4], the FCC ruled that the digital TV whitespaces be used for unlicensed access by *fixed* and *portable* devices. Fixed devices (e.g., IEEE 802.22 base stations) are used for providing last mile internet access in underserved areas, while portable devices can be used to provide short range wireless connectivity for Internet access (e.g., Wi-Fi like access points). Furthermore, the FCC ruled that portable devices could only transmit in channels 21-51 (i.e., 512-698 MHz) and use a transmit power of 40mW when adjacent to a TV channel in the frequency band and 100mW on a non-adjacent channel. FCC has imposed further restrictions on out-of-band emissions etc. that will be described in Section 3.

The FCC ruling has the potential for next wireless revolution. The short range wireless access in DTV-WS has been referred to as "Wi-Fi on steroids" and "Wi-Fi 2.0" in the media. Indeed, unlicensed access in DTV-WS can not only decrease congestion on the 2.4 GHz ISM band, but also provide much better data rates and coverage due to superior propagation properties of the spectrum.

In this paper, we perform a comprehensive design exercise of a system for Wi-Fi like unlicensed access in the DTV whitespaces in an enterprize setting. Throughout this paper, we say "Wi-Fi like" system to mean wirless LAN with *(i)* access points (APs) connected to the Internet, and *(ii)* clients who associate with the APs. Enterprize setting allows us to design a system and architecture where a central controller can be responsible for performing efficient spectrum allocation based on access point demands. Our goal is threefold: *(i)* develop a thorough understanding of the effect of frequency dependent radio propagation and out of band emissions on system design, *(ii)* derive an FCC compliant multi-radio based architecture based on this understanding and *(iii)* design algorithms to efficiently allocate variable spectrum to access points based on their demand. We assume that good spectrum sensing techniques for wireless microphones etc. are available to the radios. Moreover we also assume that the available whitespaces in a given location vary slowly over time. In other words, we do not address the issue of correctly detecting primary transmitters and accessing the whitespaces over very small timescales (e.g., [11, 27]).

1.1 Challenges and Contributions

We consider a system comprising several access points, each with multiple cognitive radios, i.e., radios with the ability to tune their bandwidth, center frequency and power. While cognitive radios increase the flexibility to efficiently operate the system, it also imposes the challenge of tuning a plethora of system knobs, including power, center frequency, bandwidth, guard band and power spectral density. This is significantly different from traditional systems, where the standardization bodies were responsible for determining center frequencies, channel bandwidths, power spectral density and guard bands, and the system planner only had to allocate power and channels so as to maximize system capacity. There has been some significant recent advancement in the field that design systems and protocols for demand based dynamic spectrum allocation in different contexts [8–10, 18, 23, 29, 30]. However they study different problems where frequency dependent radio propagation and the diverse and fragmented nature of the spectrum do not come into play. Specifically, three key aspects that lead us to a novel system design are the following:

A1. *Propagation Characteristics:* Path loss is directly proportional to the square of the carrier frequency. Specifically lower frequencies propagate much farther than high frequencies.

A2. *Out of Band Emissions:* Radio transmissions are never entirely confined to their operating bandwidth. Some power leaks into the adjacent parts of the spectrum causing adjacent channel interference. This has typically only been accounted for by standardization bodies by providing sufficient guard bands between channels, and recently in [6] to show that partially overlapped channels can be used in Wi-Fi systems.

A3. *Diverse and Fragmented Spectrum:* DTV whitespace spans a large range (180 MHz) relative to the band of operation (512 MHz-698 MHz). In several cities, DTV whitespaces are fragmented with several whitespaces only 6 MHz in width.

A1 and A3 together imply that the system cannot be modeled using a single interference graph, i.e., two devices interfering in one whitespace need not interfere with each other in another whitespace. Therefore doing suitable dynamic allocation of spectrum requires the precise knowledge of the interference graph in different whitespaces especially because the whitespaces are spread across a large range of spectrum. Clearly the interference graph in a whitespace can be learnt using interference measurements over that whitespace, but performing such measurements over every whitespace can incur a lot of overhead when the spectrum range is large. A key challenge we address is to learn these interference graphs with minimal overhead. We also note that for a given transmit power and range, lower parts of the spectrum have higher spectral efficiencies (bps/Hz). However, assigning lower part of the spectrum to an AP also increases the interference. A key challenge is to design demand based spectrum allocation algorithms that accounts for this trade-off. The algorithmic challenge is further augmented by the fact that the third facet calls for each access point to have multiple radios. This is primarily due to A3 and other reasons to be explained in Section 5.

A1 and A2 imply that if two devices in the vicinity are transmitting in adjacent parts of the spectrum, then the guard band required between these two parts of the spectrum is a function of the carrier frequency and transmit power. Note that this guard band is required to minimize adjacent channel interference. Thus unlike traditional systems, the system designer cannot ignore this dependence in both architecture and algorithmic design.

Our main contributions are as follows:

1. We abstract out four core design rules that capture the essence of the interactions between the different factors. Our first two rules determine what transmit power to use and what guard band to use. Our third rule illustrates how interference measurements over one part of the spectrum can be used to infer interference over the entire spectrum. For the fourth design rule, we propose a single metric (called *aggregate spectral efficiency*) that, for each AP, captures the relationship between achievable data rate, spectrum allocated, and client-locations. See Sections 3, 4.
C2. Based on the design rules, we design a mutli-radio based system architecture for Wi-Fi like unlicensed access in DTV whitespaces with carefully chosen fixed transmit power. Specifically we use a single control channel that serves two purposes *(i)* to establish communications with clients, learn the client's demand and compute the aggregate spectral efficiency, and *(ii)* to make interference measurements with other access points, which can be translated into interference graphs for different whitespaces. We make no assumptions that there is no spatial variation in the whitespaces that different access points see. See Section 5.

3. We develop algorithms for demand based dynamic spectrum allocation in DTV-WS for guaranteeing *proportional fairness*, i.e., for maximizing overall log-utility of the system. Our algorithms provide constant-factor approximation guarantee when the interefernce graph *at every frequency* is a disc graph. Our algorithms account for spatial variation in the whitespaces seen by different access points. See Section 6.

4. Via simulations we show that, on an average, our algorithms perform within 94% of the optimal fair allocation in typical scenarios. We also capture the performance improvement over dynamic spectrum access in the 2.4 GHz ISM band and show that DTV-WS can provide significant improvements over the optimal allocation in the 2.4 GHz band. Thus, short range unlicensed access in the DTV-WS is indeed *Wi-Fi on steroids*. See Section 7.

2. RELATED WORK

Spectrum Allocation: The KNOWS [23, 29, 30] project at Microsoft has developed a hardware prototype, a carrier sense MAC and algorithms for dynamic spectrum access. The platform also uses two radios, one to scan the spectrum for detecting white space and another for subsequent data communications. While related to our work, factors such as propagation characteristics, co-channel interference and out-of-band emissions, do not come into play for their system design.

In [21], the authors consider a problem very similar in nature to ours. However, in this paper, issues of out-of-band emissions and frequency dependent interference graphs do not come into play. This increases the complexity of the problem we address significantly. In [7], the authors design and implement a Wi-Fi-like system with Wi-Fi components that operates over UHF whitespaces. [7] also demonstrates how spectrum fragmentation and spatial variation of spectrum have implication on network design.

Among other work, where propogation characterstics and out-of-band emissions are not critical, notable are [8,9,18] for dynamic spectrum allocation in a coordinated fashion for cellular networks.
Others: [10] looks at a key design issue in dynamic spectrum access - namely adaptively varying the bandwidth of a radio and its impact on its throughput, range, and power consumption. This uses commodity 802.11 hardware. However since this is in the context of the 2.4 GHz ISM band several of the issues we consider do not come into play here. Another recent paper [15] proposes using spread spectrum codes to perform demand based bandwidth allocation to the different access points. Once again, the context of this paper is different from the problem we wish to address.

Finally [20] provides an alternative perspective on the amount of whitespace available.

3. BACKGROUND

In this section, we review aspects of radio propagation, out-of-band emissions, FCC regulations and DTV whitespaces, which will be crucial for later discussions.
Radio Propagation: In free space, the received power at a distance d from a point source radiating at wavelength λ with power P_t and antenna gain G is $P_r = GP_t(\lambda/4\pi d)^2$ [26]. To account for additional phenomenon that occur in the real-world, such as, reflection, diffraction and scattering, the ITU has recommended the following path loss model in the range 900 MHz-100 GHz: [1]

$$PL(d) = 10 \log_{10} f_c^2 + 10\eta \log_{10} d + L_f(n) - 28 \text{ (in dB)}, \quad (1)$$

where f_c is the carrier frequency, η is the path loss coefficient, $L_f(n)$ is the additional loss due to the number of floors n between the transmitter and receiver. Thus, there is an f_c^2 dependence of the path-loss on the frequency. Such a dependence has also been experimentally verified for frequencies as low as 450 MHz [5], and also for outdoor environments [2]. Some recent studies have also shown that the frequency dependence is larger than a square law in some cases [19], however for this paper we will assume an f_c^2 dependence which we believe to be largely true.

REMARK 1. *While this appears to be a remarkable result, we do not claim that measurements in one frequency band (say 10MHz), reveals all information propagation effects about propagation in a much higher band (e.g., 10GHz). This is because material related properties such as absorption etc., vary with frequency. However, we believe that this relationship holds true across a few hundred MHz.*

Out of Band Emissions (OE): Signals transmitted by a radio are never entirely confined within the intended bandwidth B. Out-of-band emission (OE) is the signal energy that leaks outside the main band B[1]. Regulatory authorities and standardization bodies typically prescribe a *spectrum mask* for any technology in a particular band of operation. The mask specifies the rate at which the power spectral density (PSD) decays relative to the peak power spectral density outside the intended band of transmission. For example, the IEEE 802.11g standard specifies the PSD decay at a rate of 1.1 dB/MHz, outside the main band, as shown in Fig. 1.

The OE of a transmitter, at a frequency f MHz away from the edge of a B Hz wide transmission spectrum is given by

$$OE(f) = \frac{P_t L}{B} e^{-\alpha f} \text{ (in W/Hz)} \tag{2}$$

where, L and α are constants that depend on the spectrum mask. For example, $L = -20$ dBr, $\alpha = -1.1$dB/MHz [2] for the IEEE 802.11g spectrum mask (Fig. 1). Therefore the measured emissions outside the intended band will be determined by the *peak power spectral density,* L and α specified by the spectrum mask [3].

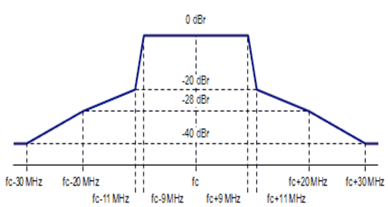


Figure 1: Spectral Mask for IEEE 802.11g/a

Adjacent Channel Interference (ACI): The spectrum mask allows us to compute the adjacent channel interference precisely. For example, assume that two devices are separated by a distance d in free space. Furthermore, assume that the first device is transmitting at power P_t over a bandwidth B in frequency band f_c. Assume that the second device is receiving over a bandwidth of W, and that there is a guard band of δ between the two bands. Then the total adjacent channel interference can be computed by integrating the leakage power over the adjacent channel of bandwidth W, using Eqn. 2 and Eqn. 1 with $L_f(n) = 0$ as

$$\frac{P_t L 10^{2.8}}{B d^{\eta} f_c^2 \alpha} e^{-\alpha\delta}(1 - e^{-\alpha W}) \tag{3}$$

FCC Regulations: After the transition to all digital TV transmissions, the available TV bands are $54 - 72$ MHz (channels $2 - 4$), $76 - 88$ MHz (channels $5, 6$), $174 - 216$ MHz (channels $7 - 13$) and $470 - 698$ MHz (channels 14-51). In its recent *Second Report and Order* [4], the FCC has ruled that these bands can be used for unlicensed access of *fixed* and *portable* devices. We highlight below the regulations that are relevant to the problem we will be addressing.

Channel Occupancy Database: The FCC requires the maintenance of a Channel Occupancy Database (COD) which registers locations of TV transmitters, the channels they operate on and their service areas. In addition the database is also required to register public venues such as entertainment centers and sporting venues to avoid interference with wireless microphones which may be used at these locations.

Fixed Devices: Fixed devices can operate from channel 2-51 and are more relevant to the IEEE 802.22 standard and hence are not discussed here.

Portable Devices: Portable devices are intended to provide short range communications similar to Wi-Fi, and they can operate in two modes: either as a client (Mode I) or independently (Mode II). In Mode I, the portable device is in a master slave relationship either with a fixed device or a Mode II portable device. A Mode II device (similar to a Wi-Fi access point), should have geo-location ability and the ability to sense wireless microphones/TV signals up to -114 dBm. Portable devices can operate only in channels $21 - 51$. A portable device can transmit at a maximum power of 40 mW (resp. 100 mW) when operating in a whitespace adjacent (resp. non-adjacent) to a TV channel.

The FCC specifies that the *out of band emissions from an unlicensed white space device be at least* 55*dB below the highest average power in the* 6*MHz channel adjacent to the* 6*MHz channel in which the unlicensed device is operating*.

Interference to DTV Receivers: The FCC requires that in the worst case, a white space device should not cause interference to a DTV receivers due to both co-channel interference or out-of-band emissions (adjacent channel interference). The ATSC standard [3] for TV receivers specifies a SINR value of -15.5dB. Therefore to protect even weak TV signals (i.e., received signals at -83.5dBm), the interference level should be below -99dBm. Since the location of the DTV receiver is unknown, the FCC report (see para 235 in [4]) argues that it is reasonable to assume that the DTV receiver is 10m away in another premise behind a wall. This results in an additional 18dB loss. Consequently, from Eqn. 1, we will assume that the path loss to the DTV receiver at distance d is given by

$$PL_{dtv}(d) = 10\log_{10} f_c^2 + 10\eta\log_{10} d - 10 \text{ (in dB)} \tag{4}$$

Analysis of DTV Whitespaces: A study conducted [14] shows that there is sufficient whitespace available. Summary statistics of the available whitespace in various markets is shown in Table 1.

4. DESIGN RULES

In this section, we distill four design rules to *(i)* determine transmit power, *(ii)* determine guard band between adjacent slices of the spectrum, *(iii)* show how interference graphs in different whitespaces can be learned with minimal overhead and *(iv)* devise a metric *aggregate spectral efficiency* (units are bits/sec/Hz) that captures the average data rate that each AP can get from unit bandwidth in

[1]The dominant source of out of band emission is the non linearity of power amplifiers. Other sources are imperfect receiver filters, oscillator radiation and interference from image frequencies

[2]Note that, α in dB/MHz = $10\log(e^{-\alpha})$.

[3]We note that, for certain spectrum masks, the OBE does not fall linearly outside the bandwidth of interest, rather it falls in a staircase like manner. However, (2) is typical of OFDM systems (eg., 802.11g spectrum mask) and we will use this to derive our insights.

Table 1: Available DTV-WS in some US urban and rural areas [14]. LOW is 54-88, MID is 174-216 and UP is 470-698 MHz band. The minimum size DTV-WS is 6 MHz everywhere.

Market	Total available WS(MHz):			Max WS
	LOW	MID	UP	size
Boston	30	36	84	36
Dallas	30	36	72	24
San Francisco	30	36	78	30
Seattle	24	36	120	30
Trenton	30	30	90	30
Little Rock, AR	24	30	138	30
Juneau, AK	12	12	198	84
Columbia, SC	30	36	150	36

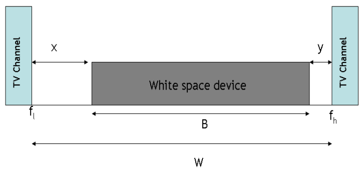


Figure 2: Figure illustrating how a whitespace could be used by a single transmit-receive pair

different whitespaces. These rules allow us to design a simple system architecture that accounts for the complex interplay between various interconnected factors.

Determining Transmit Power:

We start by considering the following simple scenario. Consider a single transmit-receive pair separated by distance d in a single DTV whitespace of width W MHz. Assume an AWGN channel with noise power spectral density N_o . We pose the following question. *What is the maximum capacity achievable in this whitespace and the corresponding bandwidth and transmit power?* We assume that the white space spans from frequency f_l to f_h, with $f_h - f_l = W$. We assume that there is a guard band of x (y) MHz between the DTV channel on the left (right) and the white space transmission to avoid ACI to DTV receivers. We assume that B MHz is used for transmission by the transmit-receive pair. See Figure 2.

The path loss between the transmit receive pair is given by Equation 1, where $L_f(n)$ is zero. We assume that the white space device has a spectrum mask with parameters L and α as in Equation 2 and that the path loss to the DTV receiver at distance 10 m is given by $PL_{dtv}(10)$ as in Equation 4. Assume that the maximum interference which can leak into the DTV receiver is given by I_{dtv}. We assume that we operate in the high SNR regime, i.e., $SNR \gg 1$. Our objective is to maximize the Shannon capacity, i.e.,

$$\max C = B\log_2\left(\frac{P10^{2.8}}{(f_l + x + \frac{B}{2})^2 d^\eta}\frac{1}{N_0 B}\right) \quad (5)$$

$$\text{subject to,} \quad \frac{10P}{f_l^2 10^\eta B\alpha} L e^{-\alpha x}(1 - e^{-6\alpha}) \le I_{dtv} \quad (6)$$

$$\frac{10P}{f_h^2 10^\eta B\alpha} L e^{-\alpha y}(1 - e^{-6\alpha}) \le I_{dtv} \quad (7)$$

$$B + x + y = W \quad (8)$$

$$x, y, B, P \ge 0 \quad (9)$$

The objective function (5), is obtained by assuming $SNR \gg 1$ with a transmit power P at carrier frequency $f_l + x + \frac{B}{2}$. The received power is computed using 1. Equations 6, 7 are obtained using Equation 2 and 4 to ensure that the total out-of-band emissions to a DTV receiver (located as per FCC recommendations) receiving TV signals in the left or right DTV channel, does not violate the interference constraints.

THEOREM 1. *The solution to the optimization problem 5 is given by:*

$$B^* = \min\left(W, \frac{1}{\alpha}\log\left(\frac{4\gamma f_l^2}{N_o(2f_l + W + z)^2}\right) + \frac{W+z}{2}\right) \quad (10)$$

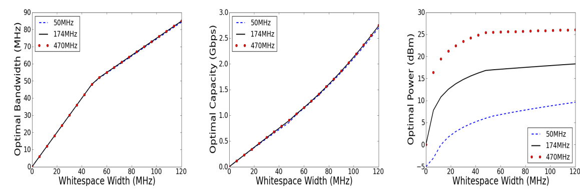
$$x^* = \frac{W + z - B^*}{2},\ y^* = \frac{W - z - B^*}{2},\ P^* = \gamma f_l^2 e^{\alpha x^*}$$

$$C^* = B^*\left[\log_2\left(\frac{4\gamma f_l^2}{N_0(2f_l + W + z)^2}\right) + \alpha x^* \log_2(e)\right]$$

where

$$\gamma = \frac{I_{dtv}\alpha}{L(1 - e^{-6\alpha})}\left(\frac{10}{d}\right)^\eta,\ z = \frac{2}{\alpha}\log_e\left(\frac{f_h}{f_l}\right)$$

PROOF. Please see [12]. □



(a) B^* vs. W (b) C^* vs. W (c) P^* vs. W

Figure 3: Optimal Bandwidth (B^*), Capacity (C^*) and Power (P^*) versus Whitespace Width (W) for $W = 50, 174, 470$ MHz, computed with a spectrum mask of $L = -50$dB and $\alpha = -2.28$ dB/Mhz.

In order to adhere to the FCC mandate of a 55 dB fall in the adjacent 6 MHz, we set $L = -50$ dBr and $\alpha = -2.28$ dB/MHz[4] and $I_{dtv} = -95$ dBm. With these values for the spectrum mask, we plot the optimal capacity, bandwidth and power as a function of the whitespace width for typical whitespace frequency bands in Figure 3. From Figure 3 we observe the following:

Observation 1: For the DTV transmission bands, the optimal bandwidth and capacity are *identical* across all frequency bands of operation from 54 MHz - 698 MHz, i.e., channels 2 - 51. This is because the restriction of adjacent channel interference to the DTV receiver, negates any benefit one obtains from moving to lower frequency bands of operation. Note that this observation is always true. However the exact optimal values depend on the spectrum mask parameters L, α and I_{dtv}. The transmit power however depends on the frequency of operation and increases with the frequency band of operation.

Observation 2: For our choice of spectrum mask parameters L and α, the optimal bandwidth is equal to the available whitespace width, up to a width of 48 MHz. Thereafter, the optimal bandwidth's dependence on W is roughly $W/2$. For white spaces larger than 48 MHz, a guard band is required between the white space transmission and the DTV transmitter when transmitting at the optimal power. The optimal power goes up to a maximum of about 27 dBm in the 500 MHz range, i.e. channels 21 - 51.

[4]This spectrum mask is used throughout the paper for illustration. Any valid choice of L and α has to satisfy $\int_0^6 OE(f)df \le P_t 10^{-5.5}$, where $OE(f)$ is given by Eqn. 2 with $P_t = 40$ mW and $B = 6$ MHz.

Observation 3: Recall that the FCC mandate gives two alternative transmit power limits: either 40 mW over the entire whitespace, or 100 mW with a guard band of at least 6 MHz between operating spectrum of AP and DTV channel. From Figure 3, when $B^* = W$ (with our parameters, for $W < 48$ Mhz) the optimal transmit power is P^* which can be as high as 27 dBm in channels 21 - 51; however, since B^* is the entire whitespace, the FCC mandate imposes a power cap of 40 mW. When $B^* < W$ (for $W > 48$ MHz), even though the optimal transmit power could be higher than 100 mW, operating in a non-adjacent channel imposes a power cap of 100 mW by the FCC mandate. Our computations show that, the capacity loss due to FCC imposed power caps is less than 10% in the first case, and less than 6% in the second case. Thus, the FCC's recommendations for 512-698 MHz are almost optimal in terms of capacity for any whitespace width.

Observation 4: For typical cognitive radio systems, there is a maximum bandwidth limit B_m across which a radio can operate ($B_m = 40$MHz with many current technologies, e.g., GNU radio, IEEE 802.11n). Assume that you are operating in an non adjacent channel at 100 mW over a bandwidth B_m. Then the difference in capacity between using 100 mW and 40 mW is only 7.5% when $B_m = 40$MHz.

In summary, we do not lose much in terms of capacity by fixing the transmit power at 40mW. Fixing the transmit power at 40 mW simplifies the design space considerably and more importantly increases spectrum reuse in the system as compared to 100 mW transmit power.

The FCC recommends a -55 dB loss in the adjacent 6 MHz channel when the whitespace device transmits at 40 mW in a 6 MHz channel. The spectrum mask is determined by this requirement. Once the spectrum mask is fixed, the out-of-band emissions are determined by the power spectral density (see Equation 3). Thus an operating bandwidth much lower than 6 MHz with a transmit power of 40 dBm adjacent to a DTV channel will have too high a power spectral density, resulting in out of band emissions that violate the FCC's requirements.

Motivated by the discussion so far and to simplify design, even when multiple pairs share the same whitespace width W (each using possibly different parts of the whitespace), we propose the following design rule:

DESIGN RULE 1. *Consider a white space W, which is to be shared between several APs. Ensure that each AP gets at least* 6*MHz and set the transmit power for each AP to 40 mW.*

Note that the rest of the paper does not critically depend on this design choice of 40 mW. All we suggest is that power control is not done. There are more complex choices possible if transmit power control is allowed. However, this significantly increases the complexity of the design space.

Guard Band:

In the following, we assume each AP can provide coverage up to distance d. To compute the worst case ACI, we assume that two clients, A and B are communicating with two radios r_A and r_B respectively (They need not be associated with same AP). Client A is located at the edge of the transmission range of the r_A. We consider the following simultaneous transmissions: r_A receiving from Client A over bandwidth B_m and r_B transmitting to Client B over a bandwidth of 6MHz. These bandwidths are chosen so as to maximize the ACI (see Equation 3). Our goal is to compute the guard band for two cases: *(i)* the two radios r_A and r_B belong to the same access point and *(ii)* when r_A and r_B belong to different APs separated by a distance d_{aci}.

For the first case successful reception at r_A requires a minimum SINR threshold of γ:

$$\frac{\frac{P10^{2.8}}{d^\eta f_c^2}}{\frac{PLe^{-\alpha x}(1-e^{-\alpha B m})}{6\alpha} + N_0 B_m} \geq \gamma$$

$$\Rightarrow x = \frac{1}{\alpha}\log_e\left(\frac{\frac{\gamma PL(1-e^{-\alpha B_m})}{6\alpha}}{\frac{P10^{2.8}}{d^\eta f_c^2} - \gamma N_0 B_m}\right),$$

where x is the guard band. Note that beyond this threshold, the data rate depends on the actual SINR.

By setting f_c to $f_c(max) = 698$ MHz, transmit power to 40 mW, range of 50 m, a worst case $\eta = 4.5$ and $\gamma = 6$ dB, we get the maximum possible adjacent channel separation of about 20 MHz. We note that even with f_c as low as 512MHz (channel 21), the channel separation only decreases marginally. The corresponding channel separation in the 2.4 GHz ISM band increases to about 25 MHz. This validates experimental results which show that with smaller guard bands in a multi-radio Wi-Fi system, radios on the same AP can either transmit or receive simultaneously, but not do both at the same time [25].

To compute the guard band in the second case, with the same settings as above, our computations show that for d_{aci} as small as 2 m, the guard band requirement falls to zero.

Based on these computations, we propose the following design principle:

DESIGN RULE 2. *The guard band depends on the frequency of operation. For channels 21 - 51, the guard band between frequency allocations on two different radios on the same AP should be separated by at least* 20*MHz. Thus for the choice of spectrum mask parameters $L = -50$ dBr and $\alpha = -2.28$ dB/MHz, adjacent channels with no guard band can be allocated to adjacent access points*[5].

Interference Graph:

A standard technique to mitigate co-channel interference between two wireless devices (not talking to each other) is to ensure that they are allotted the same channel only if the received power from one device at the other device is less than a certain threshold β. For our problem, the path loss depends on frequency and hence two APs that interfere with each other in one part of the spectrum need not do so in another part of the spectrum. Therefore the interference graph between different APs in the system is a function of the whitespace that is being used. Note that even in the 512 - 698 MHz band, this can hold true as observed in our simulations (Section 7). This effect could be much more pronounced if the FCC allows access to portable devices across Channels 2 - 51. We now illustrate how received power measurement in one part of the spectrum can be used to infer the received the power in some other part of the spectrum. Consider two APs AP_1 and AP_2, and suppose AP_1 is transmitting using power P. Suppose $P_r(f_1)$ is the received power at AP_2 when the transmission happens using carrier frequency f_1, and similarly $P_r(f_2)$ is defined. Using Equation 1, it can be easily shown that

$$P_r(f_1) - P_r(f_2) = 20\log(\frac{f_2}{f_1}) . \quad (11)$$

Without loss of generality, assume $f_1 < f_2$. Equation 11 shows that if either of $P_r(f_1)$ or $P_r(f_2)$ is known, the other can be inferred. If each AP experiences different amounts of ambient interference in each whitespace, then following two step process can be followed. In the first step, each access point reports (via a control channel say) the ambient interference it experiences in each whitespace to a central controller (see Remark 3 in Section 5). In

[5]This is not true for IEEE 802.11g where the spectrum mask parameters are more lax with $L = -20$ dBr and $\alpha = -1.1$ dB/MHz

the second step, measurements in any one frequency band are used to estimate if the total interference (ambient plus induced interference) experienced by the AP in the other frequency band exceeds a given threshold. Finally we recommend using the lower frequency bands to estimate interference graphs in the higher frequency bands since for a given receiver sensitivity, it is easier to hear control messages in the lower frequency band.

DESIGN RULE 3. *The interference map of an AP in a higher frequency band can be inferred from the interference measurements in a lower frequency band using Equation 11 along with appropriate ambient interference measurements.*

A word of caution, the path loss need not have an f_c^2 dependence and the exponent could be higher for some frequency bands [5]. This can be easily accounted for via one time measurements prior to deployment.

Aggregate Spectral Efficiency (ASE):

We introduce a new metric called aggregate spectral efficiency. Recall our goal is to do demand based spectrum allocation to APs. However doing this efficiently requires the knowledge of how much data rate does a given amount of spectrum translate to for each AP in each whitespace. A simple solution is to do a worst case design where the data rate for a given amount of spectrum is achieved using the lowest modulation. This is equivalent to assuming that all users are at the edge of the coverage region which is clearly too conservative. The ASE metric allows us to capture the dependence between the given amount of spectrum and data rate based on *(i)* the RSSI values of all clients associated with an AP and *(ii)* the location of the spectrum.

To derive the ASE metric we need to know how the data rate depends on SINR. In the ideal case this is given by Shannon's formula, which essentially says that the spectral efficiency (bits/sec/Hz) is a constant multiplier of the SINR in dB. For many modern physical layer technologies, for a given bit error rate, the achieved spectral efficiency takes discrete values that depend on SINR thresholds. Moreover, these discrete values vary linearly with the SINR thresholds in dB. For example (see Figure 4), we performed linear regression between SINR and spectral efficiency for discrete rates of IEEE 802.16d. A similar trend can be observed for IEEE 802.11g as well. Thus, we assume that spectral efficiency varies with SINR as $a + bSINR(dB)$ for some constants a and b that depend on the physical layer technology.

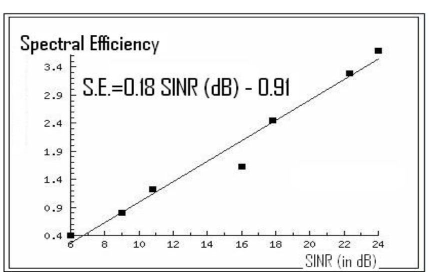


Figure 4: Result of linear regression test we performed for bits/sec/Hz vs. SINR (dB) for discrete rates of IEEE 802.16d.

Consider a carrier frequency f_1 for an AP. Assume the k^{th} client perceives $SINR_k(f_1)(dB)$. Then the spectral efficiency for the k^{th} client is $\eta_k(f_1) = a + bSINR_k(f_1)$. Assume there are N clients and each client has equal opportunity to communicate with the AP. Then the ASE is given by

$$\eta_{avg}(f_1) = a + \frac{b}{N}\sum_{k=1}^{N} SINR_k(f_1)\,.$$

This shows that in a given band, the ASE can be inferred purely from measurements of SINR. The next question is how does one obtain a good estimate of ASE across all bands in the spectrum without having to do measurements in each band. To compute this, we assume that every client experiences an average interference I_{th}. Note that the precise value of I_{th} is not required for our purpose. Consider two carrier frequencies f_1 and f_2 with $f_1 < f_2$ and let $RSSI_k(f_1)$ and $RSSI_k(f_2)$ be the received signal strengths in dB for the k^{th} client. Thus $SINR_k(f_i) = RSSI_k(f_i) - I_{th}$. Thus $SINR_k(f_2) - SINR_k(f_1) = RSSI_k(f_2) - RSSI_k(f_1) = 20\log_{10}(f_1/f_2)$, from which it follows that $\eta_{avg}(f_1) - \eta_{avg}(f_2) = 20\log_{10}(f_1/f_2)$. The preceding discussion results in the following design rule.

DESIGN RULE 4. *For any frequency band, the RSSI measurements from the clients can be used to compute the ASE. The ASE in one frequency band can be computed from the ASE in another frequency band.*

REMARK 2. *The assumption that every client sees an average interference I_{th} can be relaxed via further measurements as stated earlier.The details are left out in the interest of brevity.*

5. SYSTEM ARCHITECTURE AND SPECTRUM ALLOCATION PROBLEM

Based on the design rules illustrated in the Section 4, we describe the system architecture in this section. We start by describing the infrastructure components and their capabilities and show interaction between the different components. Finally we formulate the spectrum allocation problem. We point out that our architecture and spectrum allocation problem are general enough to capture spatial variation in whitespace availability, *i.e.*, when the availability of white spaces is not uniform across APs.

5.1 Architecture

We consider an enterprize system comprising several access points distributed across the enterprize. There are several DTV whitespaces available in the DTV band. The demand seen by the access points varies over time. The challenge therefore is to assign whitespaces to the access points based on the demand seen by the access points.

Access Points: An access point acts as a Mode II device as specified by FCC (see Section 3). Each access point comprises multiple transceivers. One transceiver is dedicated for communications over a common *control channel*. The control channel serves 3 purposes: first, it is used for client-AP association; second, make measurements to infer interference graphs over the whitespaces; and third, make measurements to compute the aggregate spectral efficiency (ASE) across all whitespaces. The control channel should operate over a frequency band lower than channel 21. We recommend this based on Design Rule 3 which implies that the interference graph in a higher frequency band can be inferred from measurements over the control channel located at a lower frequency. We also recommend that the control channel be an ISM band channel (e.g., 433 MHz) since it is available at all locations all the time. Note that the design choice of control channel with a large range in a lower frequency ISM band is deliberate, and choosing the control channel in 900 MHz ISM band (as in [**?**]) or a higher ISM band is sub-optimal for our case. All other transceivers have the ability to tune into any set of whitespace frequencies authorized by the FCC (currently channels 21-51). Note that current cognitive radio technology already allows transceivers to tune across several GHz [16]. We assume that each radio can tune into only a single contiguous band[6]. Modifying the bandwidth on demand is easily achievable in OFDM based systems and is referred to as channel bonding. See also [10] where the authors achieve this by changing the clock frequency of the PLL on commodity Wi-Fi radios.

We remark that the *medium access mechanism* by which the access point communicates with a client on any white space is left unspecified. It could be an CSMA/CA mechanism as in IEEE 802.11b/g or it could follow a scheduling mechanism as in cellular systems like WiMAX. The access point has geo-location and spectrum sensing capability in accordance with FCC rules for Mode II devices. Based on design principle 1, each access point transmits at a fixed power of 40mW.

Clients: We assume that the clients have one or more radios and acts as a Mode I device as specified by the FCC. If it has a single radio, then the client first tunes into the control channel and listens for beacon messages from the various access points. While several complex client-AP association decisions are possible, we assume that the client sends an *association request* to the access point from whom the beacon message is received with the maximum signal strength. The association request message contains its MAC ID, desired data rate (demand) and RSSI from the AP. If the association request is accepted, the access point responds with the set of white spaces, center frequencies and bandwidths over which it can communicate.

When a client is finished with its session, it sends a dissociation request to the access point. Alternatively, if an access point does not hear from the client for more than a certain amount of time, it assumes that the client is no longer associated with it.

Central Controller: The central controller has the following functionality. First, it periodically computes the interference graphs in the different whitespaces. Based on Design Rule 3, this can be performed using measurements over a single control channel. Next, for each AP, it computes the ASE in the different whitespaces based on control channel aggregate RSSI measurements provided by the AP (see Design Rule 4 for how this is computed). Finally, it computes an efficient allocation of the available whitespaces based on interference maps, ASEs, ACI constraints (see Design Rule 2), transmit power constraint (see Design Rule 1), and demands. In the next subsection, we precisely formulate this problem and develop algorithms in the subsequent section.

Finally, in Figure 5, we show the interaction between the clients, APs, and the central controller.

REMARK 3. *We note the following:*

1. *Note that the central controller can efficiently compute the interference graphs on different whitespaces by borrowing techniques for AP beacon scheduling developed in [17, 22, 24].*
2. *Performing spectrum re-allocation very frequently incurs a cost in communication overhead and switching delays. Hence the duration T should be chosen judiciously to minimize switching overheads. In addition, there could be a thresholding policy where the allocation is changed only if it is significantly different from the old allocation.*
3. *The presence of wireless microphones near APs cause non-availability of these whitespaces to APs. Also, since the wireless microphones operate intermittently, this causes temporal variation in the available whitespaces [7]. This can be handled by APs triggering a spectrum re-allocation when they detect wireless microphones. However, we believe that the time-scale of operation of wireless microphones is of the order of few minutes and thus, this would only marginally increase the overhead due to spectrum re-allocation.*

[6]Non contiguous bandwidth, via NC-OFDM is also possible, but we do not consider this in our paper since the technology is not yet mature

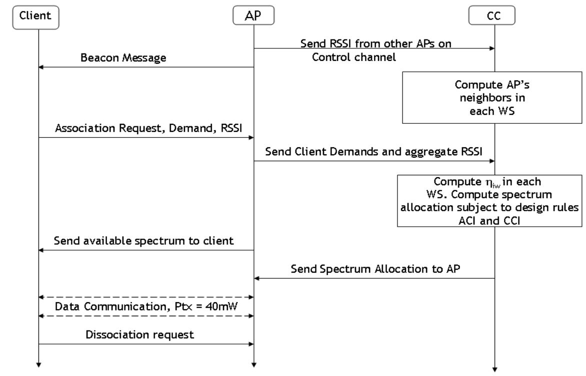


Figure 5: Timing Diagram showing interaction between clients, APs and central controller

5.2 Spectrum Allocation Problem in DTV-band

Table 2: Notations used.

Notation	Description
N_{AP}	Number of Access Points (AP's)
AP_i Acces Point no. i	AP_i
d_i	Demand of AP_i
N_{rad}	Number of radios per AP
B_m	Maximum operating bandwidth for a radio
b_m	Minimum permissible bandwidth for a radio
N_{WS}	Number of DTV White Spaces
WS_j	White Space no. j (indexed from left to right in the spectrum)
f_j	Center frequency of WS_j
W_j	Available bandwidth in WS_j
Δ_{ACI}	ACI guard band between two radios of an AP
d_{CCI}	CCI reuse distance
N_{ij}	Set of interfering AP's of AP_i in WS_j

There are N_{WS} distinct white spaces (WS), where the j^{th} white space has center frequency f_j and total available bandwidth W_j. We wish to distribute the white space bandwidth among N_{AP} distinct AP's. Each of the AP has N_{rad}[7] different radios. For the j^{th} white space, associated with AP_i is a set of other AP's N_{ij} (called neighbors of AP_i in white space j) that cannot transmit simultaneously with AP_i over the same spectrum on any of the N_{rad} radios.

Furthermore, associated with AP_i are two parameters: **(i)** demand in terms of data rate, denoted by d_i, and **(ii)** ASE over whitespace WS_j η_{ij}. Our goal is to assign spectrum to all/some of the radios of each AP in the different WS's. The algorithms we develop assume each radio gets contiguous chunk of bandwidth but is extensible to cases with non-contiguous allocation also.

Note that, our algorithms are generic enough to account for spatial variation in white space availability. Specifically, we can set $\eta_{ij} = 0$ if WS_j is not available to AP_i. In the following, we describe the system constraints and the optimization objective while performing the allocation.

[7]Our results easily apply when there are different number of radios with different AP's, but for ease of exposition we will assume that all AP's have same number of radios.

5.2.1 System Constraints and Objective

Before we describe the detail, it is important to delve into the several constraints that the system imposes.

Operating Spectrum Width (OSW): From the discussion in Section 4, B_m is the maximum allocable bandwidth to each radio of an access point.

Minimum Spectrum Width (MSW): To ensure that the power spectral density at the edge of the operating spectrum of a radio does not become too large, we need to ensure that the minimum allocable bandwidth is $b_m = 6$MHz.

Co-channel Reuse Constraint (CCI): The CCI constraints are determined by the interference graph computed by the central controller for each whitespace as described earlier.

Objective function: For most resource allocation problems, there is a tussle between maximizing system capacity and ensuring fairness to competing users. For example, in wireless systems like EV-DO, the objective is such that it maximizes the sum of the logarithm of data rates given to different users. On the other hand, maximizing system capacity is best utilization of system resources. We believe the choice should be left to the system designer who decides it based on the requirements and goals. In the following, we will provide algorithms for both the problem - that of maximizing system capacity and that of maximizing system *utility*.

The first objective function we consider is that of ensuring fairness. We choose the tried and tested practice of ensuring *proportional fairness*. In proportional fairness based schemes, the goal is to maximize overall system *utility* where the logarithm of the data rates are taken as a measure of utility. In such a fairness scheme, the objective is to maximize,

$$\text{Maximize} \quad \sum_i d_i \log(1 + r_i) \,, \tag{12}$$

Instead of d_i, we can chose some other weight that reflects the demand (for example, number of clients with the AP's). However, choosing d_i as the weight has the following intuitive interpretation: if r_i is increased to $r_i + \delta$ for AP_i, then the utility of AP_i increases by $\delta d_i/(1 + r_i) \approx \delta d_i/r_i$; thus, the additional utility of an additional δ amount of data rate is directly proportional to demand d_i and inversely proportional to existing data rate r_i. We have chosen the utility functions as $d_i \log(1 + r_i)$ instead of $d_i \log r_i$ to ensure that the system utility is always positive. This does not make any difference to the motivation behind considering a logarithmic utility functions and our algorithms.

We also consider the problem of maximizing system capacity. For this problem, our objective will be to maximize the total data rate across all AP's, subject to the constraint that no AP gets more than the requested demand. In, other words, if r_i is the data rate that AP_i gets as a result of the spectrum allocation, then, our goal is to choose r_i's so that the following is achieved (recall that, d_i denotes the demand of AP_i):

$$\text{Maximize} \quad \sum_i \min(r_i, d_i) \tag{13}$$

This is equivalent to maximizing the system capacity subject to the constraint that no AP gets more than the requested demand.

5.2.2 Problem Statement

We are now in a position to state our problem.

Proportionally Fair White-Space Spectrum Allocation Problem (PF-WSA):

Given: N_{AP} APs and their demands $d_i, i = 1, \dots, N_{AP}$; N_{rad} radios for each AP; N_{WS} whitespaces(WS's) and their widths $W_j,\ j = 1, 2, \dots, N_{WS}$ and center frequencies f_{c_j}; the ASE for AP_i over spectrum WS_j η_{ij}; and N_{ij}, the set of interfering APs of AP_i in WS_j; the maximum operating spectrum B_m for each radio of each AP, b_m the minimum bandwidth allocable to any radio and the minimum bandwidth separation Δ_{ACI} required between two radios of the same AP.

To find: An allocation of spectrum to the different radios of the AP's subject to the system constraints OSW, MSW, CCI, and ACI such that, if r_i is the data rate achieved by AP_i, then $\sum_i d_i \log(1 + r_i)$ is maximized.

Capacity Maximizing White-Space Spectrum Allocation Problem (CM-WSA):

The input to this problem is same as that of CM-WSA. However, the goal is to maximize $\sum_i \min(r_i, d_i)$

REMARK 4. *It is easy to see that the problem is at least as hard as computing the maximum weight independent set in graph. Clearly if there is a single whitespace of width b_m, and the ASEs are identical for all APs, then our problem reduces becomes exactly the problem of computing the maximum weight independent set by setting the demands as the weights. Thus PF-WSA is NP-hard to approximate within $n^{1-\epsilon}$ of the optimal.*

6. ALGORITHMS

In this section, we will describe our algorithm for dynamic spectrum allocations, and in the following sections we will provide provable guarantees when AP's lie in a plane. Throughout this section, we will use the notation x^+ as $x^+ = \max(x, 0)$.

6.1 Overview and Key Intuition

As noticed in some of the previous work [30], spectrum allocation problems are much generalized versions of graph coloring where each node has to be assigned interval in the frequency band. However, the PF-WSA and CM-WSA has two additional complexities. Firstly, multiple radios with each AP which makes PF-WSA and CM-WSA problems of assigning multiple disjoint (and sufficiently separated so as to not violate ACI constraint) intervals to nodes. Secondly, there is no fixed graph as the interference map giving rise to graph structure is different for different WS's. We develop algorithms that deal with these complexities in two steps: first solve the problem with single WS and single radio, and then perform a *local search* to iteratively improve the objective function. More precisely, our algorithm has the following key components:

- *Spectrum allocation for single WS and single radio per AP:* This sub-routine solves the problem when each AP is equipped with a single radio and the total width of the WS is such that we can use single interference graph for the entire WS (clearly, this is not possible for large WS's). The key intuition for this step is the fact that the problem can be solved easily for a clique: greedily assign spectrum to AP's that give higher increase in the utility per unit of spectrum. However, since identifying cliques in a graph is a hard problem, we perform the following approximation. Identify cliques formed by neighbors of a node for all nodes u, and then greedily assign spectrum by giving higher priority to those cliques that give the best improvement in the objective function. In the next subsections, we not only show how these steps can be carried out efficiently, but also show that this algorithm provides a provable constant factor approximation for disc graphs.

- *Iterative local search procedure:* To solve the general problem, we divide the entire spectrum into chunks of width B_m. With such a partitioning, in a single chunk, we will assign spectrum to no more than one radio of any AP. We will solve the general problem in two steps. First, assign spectrum to the AP's in each of the chunks from left to right using the spectrum allocation algorithm for a single WS. Next is the *local search*, that checks whether changing the allocation for any of the WS's provides improvement over the existing solution or not. We keep repeating the local search step a fixed number of times. In each of the steps, we ensure that none of the constraints of the problem are violated. In the next section, we outline the details of the algorithm.

We will first describe the algorithms for a single WS, and then we will show how the algorithms can be extended for multiple WS's.

6.2 Algorithms for Single WS

6.2.1 PF-WSA for single WS and single radio

We will develop the algorithm for a slightly more generic log utility of the following form:

$$U_i(r_i) = d_i \log(1 + (r_i - m_i)^+) \tag{14}$$

Our goal is to maximize $\sum_i U_i(r_i)$. The generic form can be interpreted in the following manner: data rates to AP_i is useful only if the data rate r_i is more than a minimum threshold of m_i; if $r_i \leq m_i$, then the log utility is zero. Our original log utility can be recovered by setting $m_i = 0$. This more generic form will be useful for us later when we develop algorithms for multiple WS's using the algorithm for single WS as a sub-routine.

We now describe our solution to the PF-WSA problem for a single while space.

Solving PF-WSA for a clique: The algorithm is shown in Algorithm *PfSpecAllocClique.* The algorithm starts by expressing the achievable log utility of an AP in terms of the bandwidth, and then quantizing the data rates in step of $\epsilon_q, \epsilon_q(1+\epsilon_q), \epsilon_q(1+\epsilon_q)^2, \epsilon_q(1+\epsilon_q)^3, \ldots$. (step 1- 4). The parameter ϵ_q can be chosen to trade-off computational complexity and accuracy of the algorithm. Then the algorithm greedily assigns spectrum to the AP's till the total WS width is exhausted (step 5- 23). At each stage of the greedy allocations, the following two requirements are ensured:

1. If an AP is assigned spectrum in a greedy stage, then assign it sufficient spectrum so that the log utility for that AP becomes $\epsilon_q(1+\epsilon_q)^r$ for some r. Also, if an AP has already been allocated some spectrum, the greedy choice just allocates enough spectrum for the utility to increase by one quantization level, *i.e.*, if the AP was getting an utility of $(1+\epsilon_q)^r$ before this greedy step, then it gets just enough spectrum for the log utility to become $(1+\epsilon_q)^{r+1}$.

2. The greedy choice is to assign spectrum to the AP for whom allocating spectrum for increasing the log utility to one of the next quantization levels is spectrally most efficient. There are two cases depending on whether an AP had been allocated spectrum till the previous greedy step or not. If AP_y was not allocated any spectrum till the previous greedy step, we compute slope(y) as

$$\text{slope}(y) = \max_{s \geq 0} \frac{\epsilon_q(1+\epsilon_q)^s}{q_y(s)} \;.$$

On the other hand, if AP_y was allocated spectrum till the previous step so that the utility was $\epsilon_q(1+\epsilon_q)^{r_y}$, then we compute slope($y$) as

$$\text{slope}(y) = \frac{\epsilon_q(1+\epsilon_q)^{r_y+1} - \epsilon_q(1+\epsilon_q)^{r_y}}{q_y(r_y+1) - q_y(r_y)}$$

where $q_y(r_y)$ is the minimum spectrum required for the utility of AP_y to become $\epsilon_q(1+\epsilon_q)^{r_y}$. The greedy choice is the allocate spectrum to AP_y for which slope(y) is maximum.

In the end, the algorithm computes the total utility achieved by

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Algorithm 1 *PfSpecAllocClique:* Prop. fair Spectrum Allocation in WS_j when the interference graph is a Clique C_G

1: **for all** nodes $y \in C_G$ **do**
2: Define the utility as a function of bandwidth by accounting for the spectral efficiency. Define

$$U_y(b) = d_y \log(1 + (\eta_{yj} b - m_y)^+),\; b \geq 6,$$

and $U_y(b) = 0$ for $b < b_m = 6$.
3: Quantize the utility as follows. Compute the bandwidth required to get an utility of $\epsilon_q(1+\epsilon_q)^r,\; r = 0,1,2,\ldots,r_{y,\max}$, where $r_{y,\max}$ depends on the demand d_y. In other words, compute

$$q_y(r) = \min\{b : U_y(b) \geq \epsilon_q(1+\epsilon_q)^r\} \;.$$

4: **end for**
5: Initialize spectrum given to AP_y as $b_y := 0$ for every $y \in C_G$. Also denote by r_y (valid only when $b_y > 0$) the bandwidth offered to AP_y in terms of what quantized level is reached by $U_y(\cdot)$.
6: Initialize used bandwidth as$UB := 0$ and $Y := C_G$. Also, W is the width of the WS.
7: **while** $UB < W$ or $Y \neq \emptyset$ **do**
8: **for all** $y \in Y$ **do**
9: Compute slope(y) as follows:

$$\text{slope}(y) = \begin{cases} \frac{\epsilon_q(1+\epsilon_q)^{r_y+1} - \epsilon_q(1+\epsilon_q)^{r_y}}{q_y(r_y+1) - q_y(r_y)} & ;\; b_y \neq 0 \\ \max_{s \geq 0} \frac{\epsilon_q(1+\epsilon_q)^s}{q_y(s)} & ;\; b_y = 0 \end{cases}$$

10: **if** $b_y = 0$ **then**
11: $\delta_y = \arg\max_{s \geq 0} \frac{\epsilon_q(1+\epsilon_q)^s}{q_y(s)}$
12: **else**
13: $\delta_y = 1$
14: **end if**
15: **end for**
16: Among nodes in $y' \in Y$, find the node y such that

$$y_m = \arg\max_y [\text{slope}(y)]$$

$\{y_m$ is the node that best utilizes the spectrum for going to the next quanta.$\}$
17: $UB \leftarrow UB + (q_{y_m}(r_{y_m} + \delta_{y_m}) - q_{y_m}(r_{y_m}))$
18: If $b_{y_m} = 0$, then set $r_{y_m} = \delta_{y_m}$; else update $r_{y_m} \leftarrow r_{y_m} + \delta_{y_m}$
19: $b_{y_m} \leftarrow q_{y_m}(r_{y_m})$
20: **if** $r_{y_m} = r_{y_m,\max}$ **then**
21: $Y \leftarrow Y - \{y_m\}$
22: **end if**
23: **end while**
24: **if** $UB > W$ **then**
25: $r_{y_m} \leftarrow r_{y_m} - \delta_{y_m}$
26: **end if**
27: Compute the total utility $TU(C_G)$ attainable by the nodes in C_G, and total bandwidth used $TB(C_G)$ by these nodes as

$$TU(C_G) = \sum_{y \in C_G, b_y > 0} \epsilon_q(1+\epsilon_q)^{r_y} \;\;,\; TB(C_G) = \sum_{y \in C_G} b_y \tag{15}$$

28: **if** $TU(C_G) < U_{y_m}(\delta_{y_m})$ **then**
29: $TU(C_G) = U_{y_m}(\delta_{y_m})$, and $r_{y_m} = \delta_{y_m}$ and $r_i = 0$ for $i \neq y_m$.
30: **end if**

Algorithm 2 *BestNeighborhoodFirst:* Greedy Algorithm for CM-WSA

1: Let G denote the graph formed by the interference maps of each nodes. Initialize $G_{\text{cand}} := G$ as the graph on which we will next perform greedy allocation of spectrum. At each step, G_{cand} only consists of AP's that are candidate for allocation.
2: **while** G_{cand} has at least one node **do**
3: **for all** $u \in G_{\text{cand}}$ **do** $\{G_{\text{cand}}$ denotes the interference graph in this $WS_j\}$
4: Initialize $TU(u)$, the total capacity attainable by the nodes in the neighborhood of u as $TU(u) := 0$.
5: Let CN_{uj} be the candidate neighbors for spectrum allocation. Initialize $CN_{uj} := N_{uj} \cap \{v : v \text{ belongs to } G_{\text{cand}}\}$.
6: **while** $CN_{uj} \neq \emptyset$ **do**
7: **for all** $v \in CN_{uj}$ **do**
8: Find nodes that belong to $CN_{uj} \cap N_{vj}$.
9: Apply $PfSpecAllocClique$ to $CN_{uj} \cap N_{vj}$ to compute **(i)** the total capacity $TU(u,v)$ attainable by the nodes in $CN_{uj} \cap N_{vj}$, and **(ii** total bandwidth used $TB(u,v)$ by these nodes[8].
10: **end for**
11: Find $v_m \in CN_{uj}$ for which $TC(u,v)$ is maximized, and assign spectrum according to $PfSpecAllocClique$.
12: Update $TU(u)$ as

$$TU(u) \leftarrow TU(u) + TU(u, v_m) \; .$$

13: Remove from CN_{uj} nodes in $CN_{uj} \cap N_{vj}$, and also neighbors of $CN_{uj} \cap N_{vj}$ (*i.e.*, nodes that have a neighbor in $CN_{uj} \cap N_{vj}$).
14: **end while**
15: **end for**
16: Allocate spectrum to the node u_m in G_{cand} that has maximum $TU(u)$, *i.e.*,

$$u_m = \arg \max_{u \in \text{ Nodes in } G_{\text{cand}}} TU(u)$$

17: Let V be the set of nodes that get some spectrum in the previous step. Now, update G_{cand} by deleting all nodes in V and their neighbors.
18: **end while**

the clique. Note that the updates in last iteration of the greedy choice is discarded if the used bandwidth exceeds the available bandwidth of the white space (Step 24. However, if the last iteration alone contributes to the utility more than the other iterations combined, we allocate bandwidth only to the greedy choice in the last iteration (step 28).

Solving PF-WSA for a General Interference graph: The solution of PF-WSA in the case of general graphs is shown in Algorithm *BestNeighborhoodFirst*, and has the following key stages.

Stage-1: Computing the total utility in the neighborhoods: This is shown in one iteration of the *for loop* in step 3 because this step is repeated. In this stage, we first compute the total system utility $TU(u)$, $\forall\, u$ achievable by the neighborhoods of the different nodes. We do this as follows for the neighborhood of u (step 5-14):

- For every node v that is a neighbor of u, we consider the set of nodes that are neighbors of both v and u, *i.e.*, we consider the set $N_{uj} \cap N_{vj}$. Next, we treat this set $N_{uj} \cap N_{vj}$ as a clique and use Algorithm *PfSpecAllocClique* to find the total attainable utility $TU(u,v)$. The algorithm then finds the node $v_m \in N_{uj}$ for which $TU(u,v)$ is maximum, and the corresponding $TU(u, v_m)$ is added to the existing value of $TU(u)$.

- We remove the following nodes from consideration: the node v_m, its neighbors (that also belong to N_{uj}) who get allocation in the previous step, and neighbors of all nodes who get allocation. The previous step is then repeated within the neighborhood of u, till all neighbors of u are exhausted.

Stage-2: Allocating spectrum to the best neighborhood: As shown in step 16, we then find the node u for which $TU(u)$ is maximum, *i.e.*find the node u_m whose neighborhood produces the maximum value of total utility. *Actual spectrum allocation is then done to the nodes in N_{uj} in accordance with the previous stage.*

Stage-3: Repetition of the steps: All the nodes that get allocation are removed from consideration, and also removed are their neighbors (step 17). The algorithm then repeats the first two stages on the new graph, and enters one more iteration of the *for loop* in step 3.

The computational complexity of the algorithm is

$$O(N_{AP}^4 \log(W_{max})/\log(1+\epsilon_q)),$$

where W_{max} is the maximum white space width and ϵ_q is the quantization step size.

The performance guarantee of Algorithm *BestNeighborhoodFirst* is very difficult to prove for arbitrary topologies. However, our problem can be viewed as a much more complex version of finding maximum weight independent sets (MWIS) in a graph. While MWIS is hard to approximate in general graphs, good approximations are known for special class of graphs known as disc graphs. The questions we ask is the following: for the case of disc graphs, is there any provable approximation guarantee for our algorithm? In the following, we will show that, if AP's lie in a plane, then the performance of Algorithm *BestNeighborhoodFirst* is within a constant factor of the optimal. We have the following result.

THEOREM 2. *Suppose the interefrence graph at every frequency is a disc graph, i.e., the interfering AP's of any AP_i lie in a disc around AP_i. Also, suppose TU is the total utility computed by Algorithm BestNeighborhoodFirst, and OU is the optimal utility of PF-WSA for single WS and single radio per AP. Then, there is a constant, $\psi > 1$, independent of the input to the problem such that the following holds:*

$$OU < \psi \,.\, TU$$

We refer the reader to the Appendix for the proof.

Since, AP's could very well lie in a plane for many practical scenarios, the result of Theorem 2 is encouraging.

REMARK 5. *There is a note of caution in interpreting the result of Theorem 2. The purpose of Theorem 2 is not so much of precise characterization of the worst case behavior of our algorithm, it is to rather show that the performance of our algorithm can be off from the optimal at most by a constant factor.*

We will demonstrate in Section 7 that for realistic scenarios, the performance of the algorithm is close to optimal .

6.2.2 Algorithm for CM-WSA

Again we consider a slightly more general version where we want to find data rate r_i's that maximize $\sum_i C_i(r_i)$ where

$$C_i(r_i) = \min((r_i - m_i)^+, d_i) \; . \tag{16}$$

[8]Note that, in reality, $CN_{uj} \cap N_{vj}$ need not form a clique, but we will show later that treating it like a clique does result in good approximation.

We consider WS_j of width W_j.The algorithm for CM-WSA for single radio single WS is along similar lines of PF-WSA, *i.e.*, in the first step obtain the solution for a clique, and then, use the clique algorithm as a sub-routine to solve the problem for generic interference graphs. The algorithm for CM-WSA differs from PF-WSA only in the first step of assigning spectrum for a clique, and the second step is identical to that of PF-WSA.

Solving CM-WSA for a clique: When the interference graph is such that every AP interferes with each other, our algorithm for CM-WSA greedily assigins bandwidth to the AP with maximum spectral efficiency in every step. The algorithm first initializes all AP's as candidate AP's, and also initializes available bandwidth ABW as $ABW := W_j$. The algorithm then proceeds greedily, by performing the following steps in each greedy iteration:

1. *Step-1:* For every candidate AP, $AP_i, i = 1, 2, \ldots$, the algorithm computes the effective spectral efficiency of AP_i, ese(AP_i) using the following two steps: *(i)* Find, $b_{i,max}$, the bandwidth required in WS_j for AP_i to meet its demand. Clearly, $b_{i,max} = (d_i + m_i)/\eta_{ij}$. *(ii)* Compute ese($AP_i$) as

$$\text{ese}(AP_i) = \min\left(\frac{d_i}{b_{i,max}}, \frac{C_i(\eta_{ij}ABW)}{ABW}\right)$$

The computation essentially accounts for the fact that W_j could be less than $b_{i,max}$.

2. *Step-2:* Among the candidate AP's, greedily pick the AP_y with maximum value of ese(AP_y) and allocate $b_y = \min(b_{i,max}, ABW)$ bandwidth to the greedy choice AP_y. Update ABW as

$$ABW \leftarrow ABW - b_y \ ,$$

and remove AP_y from the list of candidate AP's.

3. *Step-3:* Repeat the *Step-1* and *Step-2* till all AP's in the clique are exhausted or $ABW = 0$.

We skip the details of the algorithm for want of space.

Solving CM-WSA for a General Interference graph: The algorithm for CM-WSA for general interference graph is similar to that of PF-WSA using Algorithm *BestNeighborhoodFrist*, except that the equivalent CM-WSA algorithm for a clique is used as a subroutine instead of Algorithm *PfSpecAllocClique*

We can prove a result similar to Theorem 2 for this case as well.

6.3 Multiple White Space and Multiple Radio

6.3.1 Algorithm for PF-WSA

We now focus our attention on the main problem of solving PF-WSA for multiple WS's and multiple radios per AP. This is quite different from the problem in single WS and single radio per AP because, unlike the problem in single WS, in this case the AP's can be assigned N_{rad} distinct contiguous WS's as long as the spectrum chunks allocated to an AP respect the ACI constraint. Nevertheless, we will show that the algorithm for single WS can be used to develop an algorithm for the more generic problem. The key idea here is the idea of *local search* where we iteratively improve upon the solution by by improving the solution for individual WS's.

The idea of *local search* has been applied to other NP-hard combinatorial problem like k-median and *maximum generalized assignment problem* (GAP) [13]. Our *local search* approach is motivated by the GAP problem. The maximum GAP problem is as follows: given bins and items, value of packing item-i in bin-j, and a packing constraint for each bin that only allows certain subset of items for each bin, the GAP problem is to find an assignment that maximizes the total value of the packed items. In [13], among other things, the authors have shown that, if the problem for single bin can be solved within a constant factor of the optimal, then the GAP problem can also be solved within a constant factor of the optimal by using the idea of *local search*. The details are fairly technical and can be found in [13]. At a high level, the approach in [13] is iterative where every iteration of the algorithm finds a bin such that, if the items of that bin *alone* are rearranged, then the solution improves more than doing such a rearrangement for any other bin. We will show that, such an approach can be adapted to our problem. However, it is important to note that our problem is much more complex than GAP because spectrum is divisible and cannot be viewed as item. Furthermore, multiple radios and the non-linear objective function make our problem richer.

For ease of exposition, we will describe the algorithm for the case when $W_j \le B_m$, *i.e.*, when the white space widths are no

Algorithm 3 *PfMultWSMultRadio:* Prop. fair Spectrum Allocation

Initialize the bandwidth allocation. Denote by r_{il} the data rate AP_i gets in WS_l, and R_i as the total data rate given to AP_i.

$$r_{il} := 0, \ R_i = \sum_l r_{il}$$

repeat
  for $l = 1$ to N_{WS} **do**
    Compute, n_{il}, the total data rate to AP_i in the ACI causing whitespaces of WS_l as

$$n_{il} = \sum_{\text{lsACI} \le j \le \text{rsACI}, j \ne l} r_{ij} \ ,$$

    where lsACI (rsACI) is the leftmost (rightmost) ACI causing whitespace of WS_l.
    Define m_{il} as the threshold above which data rate to AP_i is useful in WS_l as follows:

$$m_{il} = \begin{cases} n_{il} & \text{if } n_{il} \ne 0 \\ 0 & \text{else if } |\{l : r_{il} > 0\}| < N_{rad} \\ \min_j r_{ij} & \text{else} \end{cases} \qquad (17)$$

    Define the following log-utility for AP_i

$$\begin{aligned} U_i^{(l)}(x_{il}) =& d_i[\log(1 + R_i - r_{il} + (x_{il} - m_{il})^+) \qquad (18) \\ & - \log(1 + R_i - r_{il})] \\ =& d_i \log\left(1 + \tfrac{(x_i - m_{il})^+}{1 + R_i - r_{il}}\right) \ . \qquad (19) \end{aligned}$$

    Run *BestNeighborFirst* to maximize $\sum_i U_i^{(l)}(x_{il})$. Let TU_l be the total utility as computed by *BestNeighborFirst* .
  end for
  Find the WS which gives the best improvement,*i.e.*, find l^* such that

$$l^* = \arg\max_l \left[TU_l + \sum_i d_i \log(1 + R_i - r_{il})\right] \ .$$

  Reallocate spectrum to WS_{l^*} according to spectrum allocation produced by running *BestNeighborFirst* for maximizing $\sum_i U_i^{(l)}$.
  Perform the following steps:
   1. For every AP, remove spectrum allocation from adjacent ACI-causing whitespaces of l^*, if any.
   2. For every AP, if there was no allocation in the ACI-causing whitespaces, then remove spectrum allocation from the whitespace that provides N_{rad}th least data rate to AP (this could be zero if less than N_{rad} radios have spectrum allocation).
until the loop is executed $\Theta(N_{WS} \log(1/\epsilon))$ times

more than the maximum bandwidth a radio can support. Indeed, this is mostly the case as white space widths are no more than 30 MHz or 42 MHz for most large cities and B_m is typically larger than this (for example, $B_m = 40$ MHz for latest GNU-radios). The case in which width of some of the white spaces can be more than B_m, our algorithm first performs a pre-processing step called Procedure *DivideSpectrum* that performs the following: **(i)** partitions the white spaces with bandwidth more than B_m into *segments* of width B_m (except for the least segement) and treats each segment as a white space, and **(ii)** for each segment to be treated as a white space, it finds the white space properties like center frequency, bandwidth, and data rate for the AP's per unit of bandwidth.

We will now describe the algorithm when $W_j \leq B_m$ for all j. The algorithm is shown in Algorithm *PfMultWSMultRadio*.

Iterative local search: The key algorithm is shown in Algorithm *PfMultWSMultRadio*. Note that, since $W_j \leq B_m$, spectrum allocation within a single white space is still no different from spectrum allocation in a single WS and single radio per AP. We make use of this simple observation to iteratively improve the solution. The solution starts with simply allocating no spectrum to any radio of any of the AP's. One step of the iteration is shown in step 2-12. In each of the iteration the algorithm performs the following:

1. For each whitespace, we compute the improvement that can be had by reallocating spectrum in that whitespace alone. In the following, we describe how this step can be performed using the spectrum allocation algorithm for single WS. Consider the whitespace WS_l. Note that, allocating spectrum to an AP in WS_l could end up violating two constraints, namely, ACI and the constraint that no AP can be allocated more than N_{rad} distinct chunks of spectrum (we will call this maximum radio or MR constraint). Thus, allocating spectrum to an AP in WS_l would be feasible if these two constraints are not violated. Clearly, this can be achieved if spectrum reallocation in WS_l is followed by removal of allocation from whitespaces that violate ACI and MR constraints. To this end, we define variables m_{il} that denote the amount of data rate that has to be deducted from current (*i.e.*, at the end of the previous iteration) data rate to AP_i so that allocation in WS_l to AP_i does not violate ACI and MR constraint. The computation of m_{il}'s is shown in step 4-5. Essentially, in this step, we minimize the reduction in data rate of AP_i so that allocation in WS_l does not violate ACI and MR. Step 6 defines the objective functions that captures the best reallocation we can perform in WS_l. To understand this, first note that, if reallocation is performed in WS_l, we first need to do away with the current allocation in WS_l. Suppose the current data rate to AP_i in WS_l is r_{il} and $R_i = \sum_l r_{il}$. Thus, an additional x_{il} unit of data rate to AP_i from reallocation in WS_l results in a total data rate of $R_i - r_{il} + (x_i - m_{il})^+$ to AP_i. The term $(x_{il} - m_{il})^+$ can be explained by the fact that, if $x_{il} < m_{il}$, then it is better to not perform reallocation in WS_l. Thus the improvement (as compared to allocation left after doing away with the allocation in WS_l from the previous iteration) in log utility of AP_I that can be had from the reallocation is
$$U_i^{(l)}(x_{il}) = d_i[\log(1 + R_i - r_{il} + (x_{il} - m_{il})^+) - \log(1 + R_i - r_{il})]$$
This explains step 6. In the next step, step 7, we use Algorithm *BestNeighborhoodFirst* to maximize $\sum_i U_i^{(l)}(x_{il})$. The output of the previous step is the improvement that can be had from the reallocation and we denote this by TU_l.

2. In step 9-11, we find the whitespace where reallocation results in maximum improvement in the total utility, and then, do re-allocation in that whitespace. We also make sure that, the ACI and MR constraints are not violated (as described in the previous paragraph) by reducing the data rate by as little as possible.

3. Finally, the iterative steps are repeated $\Theta(N_{WS} \log(1/\epsilon))$ times.

The computation complexity of the entire algorithm is captured in the following.

PROPOSITION 6.1. *The computation complexity of Algorithm PfMultWsMultRadio is $O(N_{WS} N_{AP}^4 \log(1/\epsilon) \log W_{max} / \log(1 + \epsilon_q))$, where W_{max} is the maximum width of a white space, and ϵ_q is the quantization step size in Algorithm PfSpecAllocClique.*

We skip the proof of the above which is not difficult to show from the described algorithms. We also have the following result that captures the performance guarantee of our algorithm.

THEOREM 3. *Suppose for any WS, the interfering AP's of any AP_i lie in a disc around AP_i (the disc is different for different WS). Also, suppose TU is the total capacity computed by Algorithm PfMultWSMultRadio, and OPT is the optimal utility of PF-WSA for multiple WS and multiple radio per AP. If Algorithm BestNeighborhoodFirst is ψ-approximate for some constant $\psi >$, then there is a constant ρ such that*
$$OPT < \frac{\rho}{1-\epsilon} . TU$$
with $\rho = \psi + 2 + \lceil \log(\frac{2\Delta_{ACI}}{b_m}) / \log b_m \rceil$. In the other words, Algorithm PfMultWSMultRadio provides a constant approximation guarantee.

See Appendix for the proof.

6.3.2 Algorithm for CM-WSA

The algorithm for this is no different from the iterative local search based algorithm for PF-WSA, except that each step of the iteration uses Algorithm *BestNeighborhoodFirst* for CM-WSA in single white space and single radio case (instead of using Algorithm *BestNeighborhoodFirst* for PF-WSA in single white space and single radio). Again, we can prove performance results similar to Theorem 3.

7. SIMULATIONS

In this section we perform simulations, with three objectives in mind: *(i)* evaluate the performance of the algorithm against the optimal solution in simple scenarios for the CM-WSA problem, *(ii)* evaluate the performance of our algorithms versus an upper bound for the PF-WSA problem and *(iii)* evaluate the performance benefit from doing dynamic spectrum allocation in the DTV whitespaces as opposed to doing dynamic spectrum allocation in the 2.4GHz ISM band.

We describe our simulation scenarios below. For the first two objectives we use the following scenario. We assume there are five access points distributed on a square of side 400 m. The transmit power is 40 mW. We assume there only two white spaces are available 512-524 MHz and 680-692 MHz. Note that these are whitespaces available in Philadelphia according to [14]. We assume that $I_{th} = -75$ dBm and $\beta = -87$ dBm and $\Delta_{ACI} = 20$ MHz. To compute spectral efficiency, we use the linear regression constants shown in figure 4. Path losses are computed using the multi-wall model in [28] (equation 3, pp. 47), with five light walls between any pair of APs and a per wall loss of 6 dB. We assume that the number of users associated with each access point is uniformly distributed from 0 to 50. Each user has a demand of 3 Mbps. Even in these simple scenarios we observe that the number of nodes that an access point interferes with is a function of the whitespace being used. In one case, an access point had three neighbors in the first whitespace and only one neighbor in the second.

We first compute the optimal solution using the GLPK open source LP solver and compare against the solution from our algorithm for CM-WSA. Figure 6(a) shows that our algorithm for CM-WSA performs very well compared to the optimal and is within 12% of the optimal on an average.

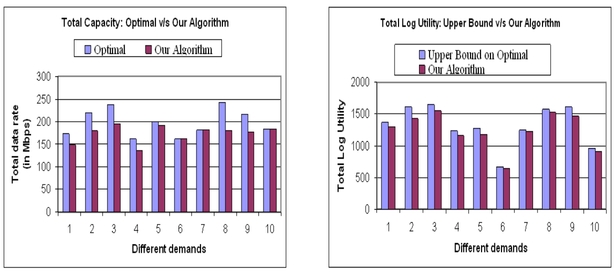

Figure 6: 6(a): our algorithm vs. the optimal for CM-WSA. 6(b): our algorithm vs. the PF-WSA upper bound.

With the same setting, in figure 6(b), we compare the performance of our algorithm against an upper bound for the PF-WSA. The upper bound can be computed as follows. Recall we are trying to maximize $\sum_i d_i \log(1 + r_i) = D \sum_i p_i \log(1 + r_i)$, where $D = \sum_i d_i$ and $p_i = \frac{d_i}{D}$. Therefore applying Jensens's inequality we have $\sum_i d_i \log(1 + r_i) \leq D \log(1 + \sum_i p_i r_i)$. Since log is an increasing function, this is optimized by maximizing $\sum_i p_i r_i$ which is a linear optimization and can be solved using GLPK for simple scenarios. We observe that our algorithm's solutions are within 6% of the upper bound on an average.

Finally, we compare the following scenario. We consider the 80 MHz ISM band in the 2.4 GHz range. This is the band used for IEEE 802.11b/g/e/n standards. For the WiFi system we set $\Delta = 25MHz$. the Correspondingly, we consider two sets of whitespaces adding up to 78 MHz (each is a subset of the whitespace available in Philadelphia and Dallas). 10 access points are distributed uniformly over a square of side 500 m. The path loss

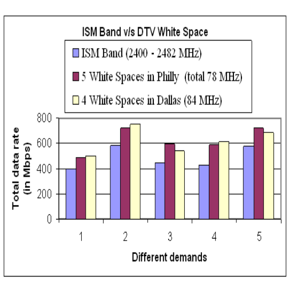

Figure 7: ISM band versus the DTV WS bands for Philly and Dallas.

models and the thresholds for I_{th} and β are the same as described earlier. Each user's demand is once again 3 MBps. In the ISM band the access points are allowed to transmit at 100 mW. We compute the optimal solution to the CM-WSA problem for the ISM band using the GLPK solver. The GLPK solver was unable to find a solution to the CM-WSA problem for the DTV whitespaces in reasonable time and hence we use our algorithm to compute the performance for the whitespaces. In Figure 7, we compare the performance in the DTV whitespaces versus the performance in the ISM band.

We observed the following interesting facts. First, for the ISM band case, none of the access points interfered with each other, while for the DTV whitespaces, several access points interfered with each other, with the number of neighbors as high as 4 in some cases. However, the spectral efficiency in the whitespaces (typically > 2.5) is much higher than the spectral efficiency in the ISM band (typically close to 1). Therefore, even though the interference graph in the ISM band is much better, the spectral efficiency in the whitespaces more than makes up for the poorer interference properties. Second, we are comparing a lower bound on performance in the DTV band with the optimal performance achievable in the ISM band with dynamic allocation. Even in this case, the DTV whitespaces out performs the ISM band by up to 42%. However note that typically the WiFi only affords fixed allocations of 22MHz in IEEE802.11b/g. Also, typically available whitespace can be much more than 80 MHz (see table 1). This clearly illustrates the fact that unlicensed WiFi access in DTV whitespaces holds significant potential as next-gen WiFi 2.0 networks.

8. CONCLUSIONS

In this paper we showed that designing systems for short range unlicensed access is a complex and challenging task. We proposed four design rules which allows us to manage this complexity. Based on these rules we proposed an architecture and algorithms for efficient allocation of spectrum to APs based on demand.

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APPENDIX

A. PROOFS OF THEOREMS

A.1 Proof of Theorem 2

To prove Theorem 2, we first prove the following lemma. The key ideas of the lemma are similar to one of the results of []. We provide the proof for the sake of completeness.

LEMMA A.1. *The total utility computed by Algorithm* **PfSpecAllocClique** *for a clique is a $2e(1+\epsilon_q)/(e-1)$ approximation of the optimal utility of the clique.*

PROOF. Consider a clique C_G that has to be allocated a total bandwidth of W. Let b_y^* be the bandwidth allocated to $y \in C_G$ under the optimal allocation. Let $U_y^Q(.)$ be the quantized version of the utilities after the quantization in step 1. Then, it is not hard to argue that,

$$\sum_y \tfrac{U_y(b_y^*)}{1+\epsilon_q} \le \sum_y U_y^Q(b_y^*) \le \sum_y U_y(b_y^*) \ .$$

Let $b_y^{(l)}$ be the amount of bandwidth allocated to AP_y after l iterations of the *while* loop. Let $z_l = \sum_y (U_y^Q(b_y^*) - U^Q(b_y^{(l)})$ and let $b^{(l)} = \sum_y b_y^{(l)}$. Let AP_j get additional bandwidth in the l^{th} iteration of the *while* loop. Note that,

$$\begin{aligned} z_l &= \sum_y (U_y^Q(b_y^*) - U_y^Q(b_y^{(l)})) \\ &\le \sum_{y:U_y^Q(b_y^*)>U_y^Q(b_y^{(l)})} \frac{U_y^Q(b_y^*) - U_y^Q(b_y^{(l)})}{b_y^* - b_y^{(l)}}(b_y^* - b_y^{(l)}) \\ &\le \sum_{y:U_y^Q(b_y^*)>U_y^Q(b_y^{(l)})} \frac{U_j^Q(b_j^{(l+1)}) - U_j^Q(b_j^{(l)})}{b_j^{(l+1)} - b_j^{(l)}}(b_y^* - b_y^{(l)}) \\ &\le (U_j^Q(b_j^{(l+1)}) - U_j^Q(b_j^{(l)}))\frac{\sum_g b_y^*}{b_j^{(l+1)} - b_j^{(l)}} \\ &\le (z_l - z_{l+1})\frac{W}{b^{(l+1)} - b^{(l)}} \end{aligned}$$

It follows that

$$z_{l+1} \le z_l(1 - \tfrac{b^{(l+1)} - b^{(l)}}{W}) \le z_0 \prod_{t\le l}(1 - \tfrac{b^{(t+1)} - b^{(t)}}{W})$$

from which it follows that

$$\begin{aligned} z_{l+1} &\le z_0(1 - \tfrac{1}{Wl}\sum_{t=0}^{l}(b^{(t+1)} - b^{(t)}))^l \\ &= z_0(1 - \tfrac{1}{Wl}b^{(l+1)}))^l \le z_0 e^{-\frac{b^{(l+1)}}{W}} \end{aligned}$$

Thus, when the *while* loop is exited with $b^{(l)} \ge W$, the approximation factor of the greedy algorithm is $1 - e^{-1}$. However, the last iteration of the *while* loop just about makes the total bandwidth exceed W, and so we undo the last iteration of the *while* loop (step 24). However, if the last iteration alone contributes more to the utility, then all the bandwidth is given to the greedy choice in the last iteration. Hence, the final utility can be reduced by an additional factor of $1/2$. The results follows. □

We next prove the following lemma.

LEMMA A.2. *Suppose the maximum utility that can be had from the AP's that belong to $\{u\} \cup N_{uj}$ be $U^*(N_{uj})$. Also suppose, the utility that can be achieved by applying Algorithm* **PfSpecAllocClique** *to $N_{vj} \cap N_{uj}$ be $U(N_{vj} \cap N_{uj})$. Then,*

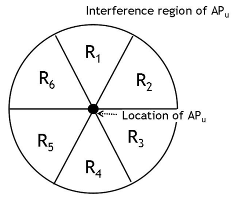
$$U^*(N_{uj}) \ \le \ \frac{12e(1+\epsilon)}{e-1} \max_{v\in N_{uj}} U(N_{vj} \cap N_{uj})$$

PROOF. First note that the AP's belonging to N_{uj} lie in a circle of radius r around AP_u, where r is the CCI distance in WS_j. As shown in Figure 8(a), we divide the circle into six disjoint regions, $R_1, R_2, \ldots, R_6$, such that each region subtends an angle 60 at the location of AP_u. Now, the neighbors of AP_u that lie in, say R_1, clearly form a clique as any two AP in R_1 lie within a distance r of each other. If $U(R_i)$ is the total utility obtained by dividing the spectrum among the AP's in R_i *alone*, then it is easy to see that,

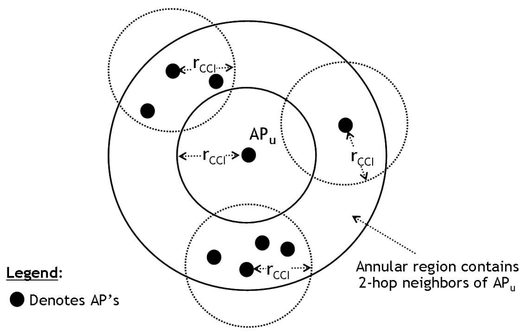
$$U^*(N_{uj}) \le \sum_{i=1}^{6} U(R_i) \le 6 \max_i U(R_i) \ . \tag{20}$$

The inquality follows from the fact that AP's in R_i, under optimal allocation, has to contend for spectrum with AP's outside R_i too, whereas, $U(R_i)$ is defined by ignoring the contention of AP's between distinct R_i's. Now, suppose $U_C(N_{vj} \cap N_{uj})$ is the optimal utility attained by treating the AP's in $N_{vj} \cap N_{uj}$ like a clique (*i.e.*, assume that no two AP in $N_{vj} \cap N_{uj}$ can share spectrum). Then, for any $v \in R_i$ for some i,

$$U(R_i) \le U_C(N_{vj} \cap N_{uj}) \tag{21}$$



(a)



(b)

Figure 8: (a) Diagram for proof of Lemma A.2. (b) Diagram for proof of Theorem 2.

since, any feasible solution for $U(R_i)$ is also a feasible solution for $U(N_{vj} \cap N_{uj})$. The result follows by combining (20), (21), and Lemma A.1. $\square$

Proof of Theorem 2: Each iteration of the *while* loop in Step 2 of Algorithm *BestNeighborhoodFirst* essentially performs a greedy iteration and in the end, removes from contention the AP's that are allocated spectrum and their neighbors in the graph. We say that AP_i belongs to iteration-l if, either AP_i is allocated spectrum in the l^{th} of the *while* loop in Step 2, or AP_i is removed from contention due to one of its neighbors getting spectrum. Thus, each AP belongs to one of the iterations.

In the following, we will argue that, for any l, the total utility of the AP's in any iteration-l can be off from the optimal only by a constant factor.

In iteration-l, the *while* loop in Step 2 first computes the utility that can be achieved from the neighborhood of every AP that is part of iteration-m for some $m \geq l$ (*for* loop in Step 3), and then, allocates spectrum to the neighborhood with highest utility (Step 16). Let u_m be the AP whose neighborhood is allocated spectrum in iteration-l. There are two kinds of AP's that are part of iteration-l: AP's that are neighbors of u_m and AP's that are not.

First consider the AP's that are neighbors of u_m. By Lemma A.2 the total utility of these AP's in the optimal solution can be better from our allocation only by a factor of $2e(1+\epsilon)/(e-1)$.

Now consider the AP's that are part of iteration-l but who are not neighbors of u_m. These AP's must be the neighbors of AP's that are allocated spectrum (see Step 17). Thus, these AP's are two-hops from u_m in the interference graph in the WS under consideration, and these lie in the annular region between a disc of radius r and a disc of radius $2r$ around u_m (please see Figure 8(b)). We will call this annular region A_{u_m} for convenience. Now choose minimum number of representative AP's in A_{u_m} such that every AP in A_{u_m} is in the neighborhood of one of the representatives[9]. For example, in Fugure 8(b), the AP's in the annular region can be covered using 3 representative AP's. Supoose there are N representatives denoted by $v_1, v_2, \ldots, v_N$. Observe that, the maximum possible value of N, the "minimum representative", is bounded from above by a constant K'. This can be argued as follows. Suppose we tile A_{u_m} using hexagonal tiles of radius $r/2$, and choose an AP (of course, if any) arbitrarily in each tile as a representative. Clearly, these chosen AP's form a valid set of representatives, and the number of representatives is bounded from above by the number of hexagoal tiles of radius $r/$ required to cover A_{u_m} which is a constant. Let $U(N_{v_i})$ and $U^*(N_{v_i})$ respectively be the total utility in the neighborhood of v_i as computed by Step 3 of Algorithm *BestNeighborhoddFirst*, and in the optimal allocation. Then, clearly

$$U(N_{u_m}) \geq U(N_{v_i}) \geq \frac{e-1}{12e(1+\epsilon)} U^*(N_{v_i}) ,$$

where the first inequality follows from the greedy choice, and the second inequality follows from Lemma A.2. We thus have,

$$\sum_{i=1}^{N} U^*(N_{v_i}) \leq \frac{12eK'(1+\epsilon)}{e-1} U(N_{u_m}) .$$

It follows that the total utility of the AP's in iteration-l can be off from the optimal only by a constant factor. The result thus follows. $\square$

A.2 Proof of Theorem 3

We will first prove that a single iteration of Algorithm *PfMultWS-MultRadio* improves the cost by a suitable factor.

LEMMA A.3. *Suppose A_n is the cost of the algorithim after n iterations. Then*

$$A_{n+1} \geq A_n + \frac{1}{\psi N_{WS}} [OPT \ - \ (2 + \psi + c)A_n] ,$$

where c is an inetger greater than $\log\lceil 2\Delta_{ACI}/b_m \rceil / \log b_m$.

PROOF. We shall first introduce some notations. Suppose, at the end of n^{th} iteration r_{il} is the data rate of AP_i in WS_l. Note that we have dropped the dependence of n on r_{il} for ease of exposition. Let $R_i = \sum_l r_{il}$ be the total data rate to AP_i after n^{th} iteration. Clearly, $A_n = \sum_i d_i \log(1 + R_i)$. Also, denote by o_{il} the data rate of AP_i in WS_l in the optimal allocation. Let $O_i = \sum_l o_{il}$ be the total data rate to AP_i under optimal allocation.

Each iteration of the algorithm essentially involves two key steps: first, for each white space WS_l finds the allocation that maximizes $\sum_i U_i^{(l)}(r_{il})$ as given in (19), and then, performs spectrum reallocation in the white space that can provide the maximum improvement from spectrum re-allocation. We will first argue the following.

CLAIM A.1. *In the n^{th} iteration of Algorithm PfMultWSMultRadio, let TU_l be the re-allocation as computed by Algorithm BestNeighborhoodFirst for the maximization of $\sum_i U_i^{(l)}(r_{il})$. Then,*

$$\sum_l TU_l \geq \tfrac{1}{\psi}(OPT \ - \ (2 + \lceil \tfrac{\log h}{\log b_m} \rceil)A_n) ,$$

where $h = \lceil 2\Delta_{ACI}/b_m \rceil$, and $\psi > 1$ is the constant factor approximation guarantee of Algorithm BestNeighborhoodFirst.

[9]These representatives form what is called a *dominating set* of the AP's in A_{u_m}

Proof of Claim: We will first consider the quantity (in the following, m_{il} is as defined in Step 5 of Algorithm *PfMultWSMultRadio*),

$$\sum_l U_i^{(l)}(o_{il}) = \sum_l d_i \log\left(1 + \frac{(o_{il} - m_{il})^+}{1 + R_i - r_{il}}\right) .$$

If $R_i = 0$, then $r_{il} = 0$ which implies

$$\begin{aligned}\sum_l U_i^{(l)}(o_{il}) &= d_i \sum_l \log(1 + o_{il}) \\ &\geq d_i \log(1 + \sum_l o_{il}) \\ &= d_i \log(1 + O_i) ,\end{aligned}$$

where, the second step follows from the inequality $\log(1 + x) + \log(1 + y) \geq \log(1 + x + y)$ for $x, y \geq 0$. On the other hand, if $R_i \geq 0$, then we have the following

$$\begin{aligned}&\sum_l U_i^{(l)}(o_{il}) \\ &= \sum_l d_i \log\left(1 + \frac{(o_{il} - m_{il})^+}{1 + R_i - r_{il}}\right) \\ &= \sum_{l:o_{il}\geq m_{il}} d_i \log\left(1 + \frac{(o_{il} - m_{il})^+}{1 + R_i - r_{il}}\right) \\ &\geq \sum_{l:o_{il}\geq m_{il}} d_i \log\left(1 + \frac{(o_{il} - m_{il})^+ - r_{il}}{1 + R_i}\right) \\ &\geq d_i \log\left(1 + \frac{\sum_{l:o_{il}\geq m_{il}}[(o_{il} - m_{il})^+ - r_{il}]}{1 + R_i}\right) \\ &\geq d_i \log\left(1 + \frac{(O_i - \sum_{l:o_{il}\geq m_{il}} m_{il})^+ - R_i}{1 + R_i}\right)\end{aligned}$$

Now consider the expression $\sum_{l:o_{il}\geq m_{il}} m_{il}$. In Algorithm *PfMultWSMultRadio*, m_{il} denotes the data rate given to AP_i in the ACI causing segements of seg_l. In case the data rate in the ACi causing segemetns is zero, then m_{il} denotes the least data rate among all the radios. Using the fact that $o_{il} > 0$ is possible at most for N_{rad} possible values of l, it can be argued that

$$\sum_l m_{il} \leq \lceil \tfrac{2\Delta_{ACI}}{b_m} \rceil R_i .$$

Letting $h = \lceil \frac{2\Delta_{ACI}}{b_m} \rceil$, we thus have

$$\sum_l U_i^{(l)}(o_{il}) \geq d_i \log\left(1 + \frac{(O_i - hR_i)^+ - R_i}{1 + R_i}\right) \tag{22}$$

If $O_i \geq hR_i$, then we have

$$\begin{aligned}&\sum_l U_i^{(l)}(o_{il}) \geq d_i \log\left(1 + \frac{O_i - (h + 1)R_i}{(h + 1)(1 + R_i)}\right) \\ &\geq d_i \log\left(\frac{(h + 1) + O_i}{(h + 1)(1 + R_i)}\right) \\ &\geq d_i[\log(1 + O_i) - \log((h + 1)(1 + R_i))]\end{aligned}$$

Using the fact that, for $c = \lceil \log(h)/\log(b_m)\rceil \geq \log(1+h)/\log(1+b_m)$, $(1 + R_i)^c \geq h + 1$, we have

$$\sum_l U_i^{(l)}(o_{il}) \geq d_i[\log(1 + O_i) - c \log(1 + R_i)] .$$

Finally, if $O_i \leq hR_i$, we have

$$\begin{aligned}&\sum_l U_i^{(l)}(o_{il}) \geq -d_i \log(1 + R_i) \\ &\geq d_i \log(1 + O_i) - d_i \log(1 + hR_i)(1 + R_i)\end{aligned}$$

Using the fact that, $R_i \geq b_m$, it can be shown that $1 + hR_i \leq 1 + \lceil \log h/ \log b_m\rceil$, and thus it follows that

$$\sum_l U_i^{(l)}(o_{il}) \geq d_i[\log(1 + O_i) - (c + 2) \log(1 + R_i)] .$$

Since, $\{i : o_{il}\}$ is a feasible solution to the problem of maximizing $\sum_l U_i^{(l)}(r_{il})$, and Algorithm *BestNeighborhoodFirst* provides a ψ-approximation guarantee, we have

$$TU_l \geq \tfrac{1}{\psi} \sum_i d_i[\log(1 + O_i) - (r + 2) \log(1 + R_i)] .$$

The result of the claim follows since $A_n = \sum d_i \log(1 + R_i)$ ■

Since the algorithm chooses the white space with maximum value of $TU_l + \sum_i d_i \log(1 + R_i - r_{il})$ (which is exactly the new value of the objective function if spectrum re-allocation is performed in WS_l), we have

$$A_{n+1} \geq \tfrac{1}{N_{WS}} \sum_l [TU_l + \sum_i d_i \log(1 + R_i - r_{il})] . \tag{23}$$

Note that,

$$\begin{aligned}&\sum_i d_i \sum_l \log(1 + R_i - r_{il}) \\ &\geq \sum_i d_i \sum_l [\log(1 + R_i) + \log(1 - \tfrac{r_{il}}{1+R_i})] \\ &\geq N_{WS} \sum_i d_i[\log(1 + R_i) + \log(1 - \tfrac{R_i}{1+R_i})]\end{aligned}$$

$= (N_{WS} - 1)A_n$

where the third step follows from the inequality $\log(1-x)(1-y) \geq \log(1 - x - y)$ for $x \geq 0,\ y \geq 0$. Using the preceding along with Claim A.1 in (23), we have

$$A_{n+1} \geq A_n + \tfrac{1}{\psi N_{WS}} [OPT - (2 + \psi + c)A_n] \tag{24}$$

□

The rest of the proof of Theorem 3 is straighforward. It follows from Lemma A.3 that

$$\begin{aligned}&[OPT - (2 + \psi + r)A_{n+1}] \\ &\leq (1 - \tfrac{1}{\psi N_{WS}} [OPT - (2 + \psi + c)A_n]) \\ &\leq (1 - \tfrac{1}{\psi N_{WS}})^n OPT .\end{aligned}$$

It follows that, for $n \geq \psi N_{WS} \log(1/\epsilon)$, $OPT \leq \frac{2+\psi+r}{1+\epsilon} A_n$.